Introduction

- **Physics** is the science of observing and describing the phenomena of the physical world.

- In this course, we will introduce the basic concepts in **classical mechanics**, which is a branch of physics concerned with the motion of bodies at a macroscopic scale.

- Sir Isaac Newton is credited for nearly singlehandedly developing classical mechanics in the late 1600s. He, along with many of his contemporaries, was motivated by understanding the movement of heavenly bodies.

- The main concepts that Newton published in his *Philosophiae Naturalis Principia Mathematica* of 1687 provide the theoretical foundation for many accomplishments in engineering ranging from structural analysis to rocket science.

- The focus of this course will be in understanding the physical intuition behind Newton’s laws and developing the skills to apply these concepts to model and solve physical problems. This should make you feel more comfortable when you take Physics 7A.

- Follow-on course Physics 7B will introduce the results of work in 19th century leading to electromagnetic theory which revolutionized modern communications, and Physics 7C will describe modern physics including quantum mechanics and relativity which govern the principles underlying semiconductor computer chips and GPS satellites.
Outline

1. Units and measurements

2. One-dimensional kinematics (1 lecture)
   (a) Speed, velocity, and acceleration
   (b) Motion at constant acceleration
   (c) Freely falling objects

3. Multi-dimensional kinematics (1 lecture)
   (a) Vectors
   (b) Projectile motion

4. Newton’s Laws (1 lecture)
   (a) The three laws of motion
   (b) Free-body diagrams, normal and friction forces

5. Circular motion (1 lecture)
   (a) Circular motion and centripetal acceleration
   (b) Dynamics of circular motion

6. Work and Energy (1 lecture)
   (a) Work done by forces
   (b) Kinetic energy and work-energy principle

7. Gravitation (1 lecture)
   (a) Gravitational force
   (b) Satellites
Units

- Physics is the science of observing and explaining the phenomena of the physical world. A standard way of quantifying these observations is of utmost importance.

- Scientists have developed a standard system of **fundamental quantities**, the smallest set of units from which all others are comprised.

- The 3 fundamental quantities that we will use to describe mechanics are:

<table>
<thead>
<tr>
<th>Quantity</th>
<th>length</th>
<th>time</th>
<th>mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>Units (SI)</td>
<td>m</td>
<td>sec</td>
<td>kg</td>
</tr>
<tr>
<td>Dimension</td>
<td>[L]</td>
<td>[T]</td>
<td>[M]</td>
</tr>
</tbody>
</table>

- Any physical quantity can be written as a composite of these fundamental units. Some examples:

\[
\text{[speed]} = \frac{[L]}{[T]} \quad \text{[accel]} = \frac{[L]}{[T^2]}
\]

\[
\text{[frequency]} = \frac{1}{[T]} \quad \text{[density]} = \frac{[M]}{[L^3]}
\]

- **Dimensional analysis** is used to check if a relationship is incorrect. For example, if somebody tells you that the expression for the wavelength of a wave with propagation speed \(v\) and frequency \(f\) is \(\lambda = f/v\), you can check its dimensions:

\[
\frac{[1/T]}{[L/T]} = \frac{1}{[L]} \neq [L]
\]

Since the expected dimension for wavelength is [L], you can conclude that this equation cannot possibly be correct.
Measurements

- No measurement is perfect, and there will always be some degree of uncertainty in the quantification of a data point. A measurement without a description of its uncertainty is meaningless.

- **Significant figures** are the count of reliably known digits in a measured number. For example, 377 m has 3 significant figures, while 0.075 sec has 2 significant figures (the zeroes are simply place holders and do not represent reliably known digits).

- There is some ambiguity in numbers that end in 0. For example, the measurement 50 km/sec does not make clear whether or not the zero is known with certainty (or if the number was just rounded to 50). However, a measurement with a device with precision to 0.1 km/sec would allow for the following measurement: 50.0 km/sec. This value has 3 significant figures.

- The count of significant figures in the result of a calculation should be the same as the least significant value in the calculation. For example, $13.4 \text{ cm} \times 2.1 \text{ cm} = 28 \text{ cm}^2$. Only two significant figures could be kept in this result due to the significance of 2.1 cm.

- **Precision** is the repeatably of a measurement with a given instrument. For example, the precision of a ruler with 1 mm demarcations might measure a length of of 12.4 mm ± 0.1 mm. This ruler has a precision of 0.1 mm.

- **Accuracy** is how close a measurement is to the true value.

- Both precision and accuracy must be taken into account when estimating uncertainty in measurements.
Speed and velocity

- The **position** of an object is its location with respect to the origin.

- The **displacement** is the *change in position* of the object.

- **Velocity** is the rate of change in position over a time interval, or an object’s displacement over time. It is a relative measure between two points and therefore has a direction as well as a magnitude.

- Say we have an arbitrary function $x(t)$ that describes the position of a particle on a line as follows, where measurements of position are shown as green dots and $x$ increases in the positive direction:

- **Average velocity** $\bar{v}_{t_1,t_2}$ of an object over a time interval $(t_1, t_2)$ is its displacement over time:

  $$\bar{v}_{t_1,t_2} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$$

- Average velocity can be interpreted as the slope between two points.
• **Average speed** is a directionless (or scalar) quantity that considers *total* distance traveled regardless of direction. Consider \( x(t) \) above from \( t=0.5 \) to \( t=1.5 \). Since there is no overall displacement of the particle, average velocity \( \bar{v}_{0.5,1.5} = 0 \). However, the particle did move a nonzero amount! Average speed over an interval \((t_1, t_2)\) is expressed:

\[
\text{speed}_{t_1,t_2} = \frac{\text{distance}}{t_2 - t_1}
\]

• To calculate **instantaneous velocity**, the displacement over an infinitesimally small time interval, take the limit of velocity as the interval \((t, t + \Delta t)\) decreases to 0:

\[
v(t) = \lim_{\Delta t \to 0} \frac{x_{t+\Delta t} - x_t}{\Delta t} = \frac{dx}{dt}
\]

• Over an infinitesimally small time interval, **instantaneous speed** has the same magnitude as instantaneous velocity. This is because the total distance traveled also shrinks to become infinitesimally small. More formally,

\[
speed(t) = |v(t)|
\]

• Notice that velocity always has a direction, velocity is 0 when there is no displacement. In our one-dimensional plot of \( x(t) \), this occurs at the local maxima and minima of position.
Acceleration

- **Acceleration** is the change in velocity over a time interval.

- **Average acceleration**, or change in velocity over a distinct interval \((t_1, t_2)\), is given by the following formula:

\[
a_{t_1,t_2} = \frac{v_{t_2} - v_{t_1}}{t_2 - t_1} = \frac{\Delta v}{\Delta t}
\]

- **Instantaneous acceleration** is found by taking the limit:

\[
a(t) = \lim_{\Delta t \to 0} \frac{v_{t+\Delta t} - v_t}{\Delta t} = \frac{dv}{dt} = \frac{d}{dt} \left( \frac{dv}{dt} \right) = \frac{d^2x}{dt^2}
\]

- Notice that acceleration always has a direction. In the one-dimensional case, when velocity is decreasing, acceleration is negative; when velocity is increasing, acceleration is positive.

- An object will move at constant velocity if the acceleration is 0.
Kinematics at constant acceleration

- We will consider kinematics of the special case with constant acceleration. Let initial time $t_0 = 0$ to simplify our discussion. Also represent initial position and velocity with $x_0$ and $v_0$ respectively.

- The acceleration over the interval $(t_0, t)$ will be:

$$a = \frac{v - v_0}{t - t_0} = \frac{v - v_0}{t}$$

Solving this equation for $t$ gives an object’s velocity after undergoing constant acceleration $a$ over time $t$:

$$v = v_0 + at$$

- The average velocity during the interval $(t_0, t)$ will be:

$$\ddot{v} = \frac{x - x_0}{t - t_0} = \frac{x - x_0}{t}$$

This can be rewritten as:

$$x = x_0 + \ddot{v}t$$

Since acceleration is constant, velocity increases linearly, and average velocity over this interval is simply halfway between the two:

$$\ddot{v} = \frac{v_0 + v}{2}$$

Our final trick is to combine the last two equations to obtain an expression for position $x$ at time $t$:

$$x = x_0 + \left(\frac{v_0 + v}{2}\right)t$$

$$x = x_0 + \left(\frac{v_0 + v_0 + at}{2}\right)t$$

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$
• One final equation is useful if the time t is not known, but rather the final state of the particle (See Giancoli section 2-5 for a derivation):

\[ v^2 = v_0^2 + 2a(x - x_0) \]

• Now we have a collection of useful kinematics equations that can be used to solve a variety of problems in one-dimensional motion. Here they are in summary:

\[
\begin{align*}
\nu &= \nu_0 + at \\
x &= x_0 + v_0 t + \frac{1}{2}at^2 \\
v^2 &= v_0^2 + 2a(x - x_0) \\
\bar{\nu} &= \frac{\nu_0 + \nu}{2}
\end{align*}
\]

• The relationships between these equations can be further explored with some knowledge of calculus. Take the generic quadratic function:

\[ x = c_0 + c_1 t + c_2 t^2 \]

Since we know that \( v = \frac{dx}{dt} \) and \( a = \frac{d^2x}{dt^2} \), we can take the derivative with respect to \( x \) repeatedly:

\[
\begin{align*}
x &= c_1 + c_2 t + c_3 t^2 \\
v &= c_2 + 2tc_3 \\
a &= 2c_3
\end{align*}
\]

This generic mathematical function can have physical interpretation, provided it is describing position! \( c_1 = x_0 \), \( c_2 = v_0 \), and \( c_3 = \frac{1}{2}a \).
Gravity for falling objects

- Objects near the earth’s surface all experience uniform acceleration due to the earth’s gravitational field. *This gravitation is constant for all objects regardless of mass* if wind resistance and drag forces at high velocity are ignored.

- The source of this acceleration is the gravitational force between the earth and all objects with mass near its surface.

- A very good approximation for the **acceleration due to gravity** is \( g = 9.80\, \text{m/s}^2 \). This number does vary slightly with latitude and elevations, but these variations are very small and can be neglected for most mechanics problems.

- In the situation of a free-falling object, the only source of acceleration is due to gravity, so the kinematics equations can be used with \( a = g = 9.8\, \text{m/s}^2 \) with careful attention to how the direction of gravity is defined.
**Vectors**

- A **vector** is a quantity with a *direction* as well as a *magnitude*. Examples of vectors include velocity, displacement, and force.

- In contrast, a **scalar** is a quantity specified completely as a number with units. Examples of scalars include speed, temperature, and mass.

- Vectors are represented as arrows in physics diagrams, with their length representing relative magnitude and their orientation indicating direction.

- Graphically, the addition of vectors $\vec{A}$ and $\vec{B}$ can be represented by tail-to-tip or parallelogram methods as illustrated below:

- Negative vectors are simply vectors whose direction is completely reversed. Consider the identity $\vec{A} + (-\vec{A}) = 0$. This defines $-\vec{A}$ as the vector, when added to $\vec{A}$, sums to 0. To illustrate:
• Subtracting two vectors \( \vec{A} - \vec{B} \) is equivalent to \( \vec{A} + ( - \vec{B} ) \) as illustrated below with the tail-to-tip method:

\[
\vec{C} = \vec{A} + ( -\vec{B} ) = \vec{A} + \vec{B} - \vec{B}
\]

• Any vector \( \vec{A} \) can also be multiplied by a scalar \( c \). The product \( c \vec{A} \) is in the same direction as before, or reversed if the scalar is negative valued. In effect, this operation scales the vector's magnitude.

• Since any vector in a plane can be written as the sum of two vectors (any any vector in 3D space is the sum of three), it is very useful to resolve or decompose a vector into a sum of its components. We can decompose across unit vectors, which have magnitudes equal to 1 and directions along the coordinate axes. We will call our unit vectors \( \hat{x}, \hat{y}, \) and \( \hat{z} \) and show a decomposition of \( \vec{r}_t = x_t \hat{x} + y_t \hat{y} + z_t \hat{z} \):

\[
\vec{r}_t = x_t \hat{x} + y_t \hat{y} + z_t \hat{z}
\]
• In order to decompose a vector into its components, the projections onto the coordinate axes are taken. The projection of a vector can be thought of as the “shadow of the vector” along the axes:

![Diagram of vector components](image)

• The components of a vector as illustrated below can be found by applying trigonometry:

\[
\begin{align*}
V_x &= V \cos(\theta) \\
V_y &= V \sin(\theta) \\
\frac{V_y}{V_x} &= \tan(\theta) \\
V^2 &= V_x^2 + V_y^2
\end{align*}
\]

• That’s it! We’re now ready to work with higher dimensional motion (we’ll stick to 2D, but this method is easily generalized to a 3- or even n-dimensional space).
Projectile motion

- Now we will use the vector tools that we have developed to decompose motion into its component parts and analyze them separately.

- The position vector represents the position of a particle relative to the origin. We write the position vector for a point:

\[ \vec{r}_t = x_t\hat{x} + y_t\hat{y} + z_t\hat{z} \]

- We can also describe displacement, the change in position, as a vector quantity. The displacement vector \( \Delta \vec{r} \) between two points is defined as the difference between their position vectors as shown:

\[ \Delta \vec{r} = \vec{r}_2 - \vec{r}_1 \]

- Recall that velocity is a vector that can be decomposed into its component velocities along each direction:

\[ \vec{v} = v_x\hat{x} + v_y\hat{y} + v_z\hat{z} \]

- As you may have guessed, acceleration can also be expressed as a vector:

\[ \vec{a} = a_x\hat{x} + a_y\hat{y} + a_z\hat{z} \]
• Motion along each dimension is independent of motion along other orthogonal dimensions! This means that any trajectory can be written as the linear combination of its component parts (like a vector).

• Once the overall trajectory has been decomposed into its components, we can treat each component as a separate one-dimensional kinematics problem!

• Our problems will be restricted to motion along two dimensions. For now, there will be no acceleration in the x-direction, so the only acceleration is in the negative y-direction due to gravity.

• The choice of axis names and directions is arbitrary, but it completely excludes acceleration due to gravity from one axis (the horizontal in this case), simplifying the problem. In another coordinate system, we might have to project acceleration due to gravity onto both axes. In general, choose a coordinate system that simplifies the problem.

• The kinematics equations are therefore split between the horizontal and the vertical cases:

<table>
<thead>
<tr>
<th>Horizontal (x-direction)</th>
<th>Vertical (y-direction)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_x = 0$</td>
<td>$a_y = -g$</td>
</tr>
<tr>
<td>$v_x = v_{x0}$</td>
<td>$v_y = v_{y0} - gt$</td>
</tr>
<tr>
<td>$x = x_0 + v_{x0}t$</td>
<td>$y = y_0 + y_{x0}t - \frac{1}{2}gt^2$</td>
</tr>
<tr>
<td>$v_x^2 = v_{x0}^2 - 2gy_0$</td>
<td>$v_y^2 = v_{y0}^2 - 2g(y - y_0)$</td>
</tr>
</tbody>
</table>
Problem approach:

1. Draw a diagram of the situation

2. Choose an origin and select a coordinate system, establishing the positive and negative directions for each axis

3. List all known quantities, such as accelerations and initial velocities

4. Decompose any vector quantities into their components and organize them by axis

5. Write relevant equations for each direction and calculate required parts

6. Recompose vector quantities if necessary
Newton’s first law: inertia

- An object at rest or in uniform motion (velocity in a straight line) will maintain its state, unless it is acted upon by a net outside force.

\[ \sum \vec{F} = 0, \text{ then } \vec{v} \text{ is constant and } \vec{a} = 0. \]

- The tendency for an object to maintain its state of rest or motion is known as inertia.

- A force is a push or a pull on an object as a result of its interaction with another object. The force of object A on object B is often denoted \( \vec{F}_{AB} \).

- Net force is the sum of all the forces acting upon an object. If the sum of all forces acting on an object is zero, all forces acting on the object are balanced and the object will maintain its state.

- This also means that an object will not accelerate if the net force acting on it is zero.
Newton’s second law: dynamics

- The net force acting on an object is directly proportional to its acceleration with the constant of proportionality equal to the object’s mass. The direction of acceleration is the same as the direction of the net force.

\[ \sum \vec{F} = m \vec{a} \]

- This law gives us knowledge of the dynamics of objects, or the effect of forces on the motion of objects.

- Mass is the measure of the inertia of an object. Roughly, how difficult it is to change the object’s motion.

- A net force acting on an object with greater mass will experience a lesser acceleration than the same net force acting on an object with lesser mass.

- The Newton (N) is a unit that measures force. It is defined as the force required to accelerate a mass of 1 kg by 1 m/s². This means that 1 N = 1 kg \cdot m/s².

- In general, the dimensions of force are \( \left[ M \right] \left[ L \right] \left[ T \right]^{-2} \), and other units are sometimes used, such as the familiar pound: 1 pound = 1 slug \cdot ft/s².
Newton’s third law: reaction

• *Whenever one object exerts a force on a second object, the second exerts on the first object an equal force in the opposite direction.*

\[ \vec{F}_{AB} = -\vec{F}_{BA} \]

• An important aspect of this law is that the reaction force of the second is applied to the first as shown below. This ensures that a net force on an object is still possible!

![Diagram showing forces](image)

• Consider the situation of pulling a wagon. You are exerting a pulling force on the wagon’s handle. The handle’s reaction is a pulling force acting on you in the opposite direction with the same magnitude.
Gravity and normal forces

- The acceleration we are most familiar with so far is acceleration due to gravity. Newton’s second law implies that some force acts upon these objects to impart acceleration on them. This is known as the gravitational force $\mathbf{F}_G$ and has this form near the earth’s surface:

$$\mathbf{F}_G = mg$$

- Objects standing on the earth’s surface do not accelerate, yet they continue to experience the gravitational force. Newton’s first law implies that some force is opposing the gravitational force and is pointed upward, perpendicular to the earth’s surface. This is known as the normal force $\mathbf{F}_N$.

$$\mathbf{F}_N + \mathbf{F}_G = 0$$

- Newton’s third law implies that the surface an object is resting upon is exerting the normal force onto the object. This means that the ground exerts a force resting on objects lying on it. Likewise, the object exerts an equal and opposite force on the ground as illustrated.
Free-body diagrams

- A free-body diagram is a drawing of an object with every force acting on it. This is a useful technique to account for all the forces in a physics problem and ensure that the correct net force is calculated.

- The diagram also helps to visualize the directions of force vectors and aid understanding their decomposition into component parts.

- Consider the example of a box on an incline. The force of gravity is oriented downwards. However, the normal force exerted on the block by the incline is perpendicular to the incline, and the component of gravity accelerating the block down the incline is parallel to its surface.
Friction

- The force of friction arises when two solid surfaces are touching each other. The roughness in the two surfaces interact and exerts a force.

- The friction force exerted on your feet by the ground enables walking. Remember that your feet also exert an equal and opposite force on the ground!

- The magnitude of the friction force $\vec{F}_{fr}$ is proportional the magnitude of the normal force $\vec{F}_N$ and is directed perpendicularly to the normal force.

- The two magnitudes are related by the proportionality constant $\mu$, the coefficient of friction:

\[
F_{fr} = \mu_k F_N
\]
\[
F_{fr} \leq \mu_s F_N
\]

Notice that only the magnitudes of the vectors are described by this relation, their directions are perpendicular!

- The coefficient of friction is affected by the characteristics of the two surfaces that are interacting. For example, a rough steel box will have a different coefficient of friction with ice than with wood.

- The coefficient of friction has a different value in static conditions (no motion) than in dynamic conditions. We denote the coefficient of static friction $\mu_s$ and the coefficient of dynamic friction $\mu_k$. Pay careful attention to the dynamics of the situation before selecting which $\mu$ to use.
Uniform circular motion

- An object that experiences **uniform circular motion** is traveling at constant speed $v$ in a circle as pictured below. The object's velocity is always tangent to the circular path.

![Diagram of uniform circular motion]

- The **frequency** $f$ of the motion is the number of revolutions per unit time. Hertz (Hz) is a measure for frequency with units [rev/sec].

- The **period** $T$ is the amount of time required to complete one revolution. Frequency and period and inverses of each other:

  $$T = \frac{1}{f}$$

- If we know the radius $r$ of circular motion, we can calculate the magnitude of the object’s **linear velocity** $v$. The distance traveled in one revolution is the circumference of the circle, and the time it takes to make one revolution is the period, so:

  $$v = \frac{2\pi r}{T} = 2\pi f r$$

- **Angular velocity** $\omega$ is the rate of change in angle measured in radians/second. Therefore, it is expressed:

  $$\omega = \frac{2\pi}{T} = 2\pi f = \frac{v}{r}$$
Centripetal acceleration

- The magnitude of the velocity is constant under uniform circular motion, but its direction is continuously changing! A change in direction of motion is a change in velocity and is therefore considered an acceleration.

- This acceleration points towards the center of the circle and is known as the **centripetal acceleration** $\vec{a}_R$.

- To understand why this is so, consider the change in the object’s velocity over a short time interval:

  $\Delta \theta$ is the angle subtended during the time interval of length $\Delta t$. $\Delta \ell$ is the corresponding arc length.

  The difference $\Delta \vec{v}$ between $\vec{v}_1$ and $\vec{v}_2$ is directed nearly exactly towards the center of the circle. This is the direction of the centripetal acceleration!
Now to determine the magnitude of centripetal acceleration, consider that when $\Delta t$ is very small, so is $\Delta \theta$. This means that the arc length $\Delta \ell$ is nearly equal to the chord length, an isosceles triangle is formed by lengths $r$ and $\Delta \ell$. A geometrically similar isosceles triangle is formed by the magnitudes $|\vec{v}_1| = |\vec{v}_2| = v$, and $|\Delta \vec{v}|$ as shown below:

- The similarity of these triangles means that the following ratios hold:
  \[
  \frac{\Delta v}{v} \approx \frac{\Delta \ell}{r}
  \]
- We are interested in the magnitude of the centripetal acceleration, so take the limit:
  \[
  a_R = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \lim_{\Delta t \to 0} \frac{v \Delta \ell}{r \Delta t}
  \]
- Because the linear velocity $v$ is equivalent to $\lim_{\Delta t \to 0} \frac{\Delta \ell}{\Delta t}$, the magnitude of centripetal acceleration becomes:
  \[
  a_R = \frac{v^2}{r}
  \]
Dynamics of circular motion

- According to Newton’s second law, some force on the object must be generating this centripetal acceleration. This center-directed force is known as the **centripetal force**. Without this force, Newton’s first law tells us that the object would carry on in a straight line.

![Diagram of centripetal force](image)

- In problems of uniform circular motion, the net force in the radial direction is equal to the centripetal force:

\[ \Sigma F_R = ma_R = m\frac{v^2}{r} \]

- When a weight is swung around a string, the string’s tension is supplying the centripetal force. When a satellite revolves around the earth, the earth’s gravitational pull is supplying the centripetal force.

- What about the **centrifugal force**? This is sometimes erroneously referred to as the force *pushing outside the circle*. This force *does not exist*! The only acceleration that occurs is towards the center of the circle, so only the centripetal force can exist.

- The sensation of being pulled towards the outside or circular motion, such as a hard turn in a car, is due to your body’s tendency to maintain a linear velocity while the car surrounding you is turning.
Work

- **Work** is done when an object is displaced by a force. Work done by a *constant force* is defined as *the product of the magnitude of displacement times the component of force parallel to the displacement*.

- The unit for work is the **joule** (J), the work done by a 1 Newton translating an object over 1 meter. 1 J = 1 N \cdot m.

- To determine work, calculate the **dot product** of displacement \( \vec{d} \) and force \( \vec{F} \) (this operation finds the product of their parallel components):

\[
W = \vec{F} \cdot \vec{d} = F d \cos \theta
\]

- The dot product between two vectors is the sum of the products of their parallel components. Since the unit vectors are perpendicular to each other, the following relationships hold:

\[
\hat{x} \cdot \hat{x} = \hat{y} \cdot \hat{y} = \hat{z} \cdot \hat{z} = 1
\]
\[
\hat{x} \cdot \hat{y} = \hat{x} \cdot \hat{z} = \hat{y} \cdot \hat{z} = 0
\]

- Recall that we can write vectors \( \vec{A} \) and \( \vec{B} \) as:

\[
\vec{A} = A_x \hat{x} + A_y \hat{y} + A_z \hat{z}
\]
\[
\vec{B} = B_x \hat{x} + B_y \hat{y} + B_z \hat{z}
\]

- The dot product \( \vec{A} \cdot \vec{B} \) is thus written:

\[
\vec{A} \cdot \vec{B} = (A_x \hat{x} + A_y \hat{y} + A_z \hat{z}) \cdot (B_x \hat{x} + B_y \hat{y} + B_z \hat{z})
\]
\[
= (A_x \hat{x} \cdot B_x \hat{x}) + (A_y \hat{y} \cdot B_y \hat{y}) + (A_z \hat{z} \cdot B_z \hat{z})
\]
\[
= A_x B_x + A_y B_y + A_z B_z
\]

- Please note that the dot product is a commutative operation!
Spring force

- When a coiled spring is stretched or compressed from its original length by a displacement of $x$, it exerts a **restoring** or **spring force** with magnitude proportional to the displacement directed opposite of displacement:

  $$F_s = -kx$$

- $k$ is the **spring constant**, a measure of the spring’s stiffness. This relationship between the spring force and displacement is also known as **Hooke’s law**.

- The spring force varies with position. A greater force $F_p = k\vec{x}$ must be applied to the spring in order to compress or stretch it as $|\vec{x}|$ increases.

- In order to find the work done on a spring by a compression or stretching force $F_p$, the force must be integrated over the spring’s displacement.
• Generally this is done using the path integral from point \( a = (x_a, x_b, x_c) \) to \( b = (x_a, x_b, x_c) \):

\[
W = \int_a^b \vec{F} \cdot d\vec{\ell}
\]

Here the differential displacement \( d\vec{\ell} = dx\hat{x} + dy\hat{y} + dz\hat{z} \) and the force is \( \vec{F} = F_x\hat{x} + F_y\hat{y} + F_z\hat{z} \).

• Therefore, the work done by a force \( \vec{F}_p \) to compress or stretch a spring over a displacement \( x \) is:

\[
W = \int_0^x F_p(x)\hat{x} \cdot dx\hat{x}
\]

\[
= \int_0^x kxdx
\]

\[
= \frac{1}{2}kx^2
\]

• Notice that the direction of the displacement (compression or stretching) does not change the amount of work done!
Work-energy principle

- **Energy** is roughly the ability to do work. This is not always true, since other forms of energy such as heat energy cannot do work, but it will suffice for our simplified description of mechanics. Energy is measured with the same unit as work, the Joule.

- **Kinetic energy** $K$ is the energy of motion. Objects in motion are able to exert forces on other objects, so by this definition they have energy.

- Consider an object with mass $m$ accelerating from $v_0 = 0$ to $v_f = v$ over a horizontal displacement $d$ as pictured below:

  ![Diagram showing an object with mass $m$ accelerating from $v_0 = 0$ to $v_f = v$ over a horizontal displacement $d$.](image)

  - The net force $\vec{F}_{net}$ is acting on the object along the horizontal to accelerate it, and is given by the relationship:

    \[
    F_{net} = ma = m \left( \frac{v^2 - 0^2}{2d} \right) = \frac{1}{2} \frac{m v^2}{d}
    \]

- By definition, the translational kinetic energy $K$ is the work done to accelerate the object to its final velocity $v$:

  \[
  K = \vec{F}_{net} \cdot \vec{d} = m \frac{1}{2} \frac{v^2}{d} d = \frac{1}{2} mv^2
  \]
• Now consider a slightly more general case where the object is accelerating horizontally from an initial velocity \( \vec{v}_0 \) to a final velocity \( \vec{v}_f \) as shown below:

![Diagram showing an object moving from \( \vec{v}_0 \) to \( \vec{v}_f \) along a distance \( d \)]

• The **work-energy principle** states that *the change in kinetic energy of an object is equal to the net work done on that object.*

• The net force done on the object over this displacement is:

\[
F_{net} = ma = m \left( \frac{v_f^2 - v_0^2}{2d} \right)
\]

• The net work done on the object is:

\[
W_{net} = \vec{F}_{net} \cdot \vec{d} = m \left( \frac{v_f^2 - v_0^2}{2d} \right) d
\]

\[
= m \left( \frac{v_f^2 - v_0^2}{2} \right)
\]

• Therefore, the change in kinetic energy \( \Delta K \) is:

\[
\Delta K = W_{net} = \frac{1}{2} mv_f^2 - \frac{1}{2} mv_0^2
\]
Newton’s law of universal gravitation

- *Every point mass attracts every other point mass with a force proportional to the product of their masses and inversely proportional to the square of the distance separating them. This attractive force is directed along the line joining the two masses.*

\[ F = G \frac{m_1 m_2}{r^2} \]

Where \( F \) is the magnitude of the gravitational force, \( m_1 \) and \( m_2 \) are the interacting masses, \( r \) is the distance that separates the masses, and \( G = 6.67 \times 10^{-11} \text{N} \cdot \text{m}^2/\text{kg}^2 \) is a universal constant.

- The extremely low value of \( G \) means that the gravitational force between two everyday objects is incredibly tiny. Two 1 kg masses 1 meter apart experience an attractive force of \( 6.67 \times 10^{-11} \text{ N} \!

- We can build some intuition regarding the gravitational force by applying what we know about uniform circular motion to our observations of the moon’s motion.
• The radius of the moon’s circular path around earth is $r = 3.84 \times 10^8$ meters and it takes 27.3 days for one revolution. With some calculation, we find that the centripetal acceleration $a_R = \frac{v^2}{r} = 2.72 \times 10^{-3} \text{ m/s}^2$. This is about $1/3600$ the acceleration due to gravity on the earth’s surface!

• Amazingly, the radius of the earth 6380 km is about $1/60$ that of 384000 km, the distance from the moon’s center to the earth’s center. From this, Newton surmised that the gravitational force decreases with the square of its distance from the earth’s center.

$$ F \propto \frac{1}{r^2} $$

• Newton knew that an object’s mass affected the force of gravity. He also figured that the moon exerts a force on the earth that is equal and opposite to the gravitational force of the earth on the moon. He therefore figured that the symmetry of the force required that the magnitude be proportional to both masses.

$$ F \propto \frac{m_1 m_2}{r^2} $$

• The relationship between distances and masses were clear, so the proportionality of the gravitational force was known. All that remained was to determine the value of $G$, the proportionality constant.

• $G$ was not measured experimentally until 71 years after Newton’s death, when Henry Cavendish did so.

• Consider what chaos would ensue if the value of $G$ changed. It’s a fundamental constant of nature that is in part the way things behave the way they do!
Gravity near the earth’s surface

• Now we can revisit our assumption about $g$, the acceleration due to gravity.

• If an object with mass $m$ is close to the earth’s surface, its distance from the center of the earth is very nearly the earth’s radius $r_E$. We also know that the earth’s mass is $m_E$, so we can say:

$$mg = G \frac{mm_E}{r_E^2}$$

$$g = G \frac{m_E}{r_E^2}$$

• Because of this relation, once $G$ was measured, the mass of the earth was finally calculated ($m_E = 5.98 \times 10^{24}$ kg)!

• Gravity is also affected by the earth’s rotation. Consider the fact that an object at the earth’s equator is experiencing a uniform circular motion. A centripetal force is required to maintain this motion.

• Some of the force of gravity is used up to supply this centripetal force, so the effective acceleration due to gravity has decreased slightly. The value for $g$ at the equator is close to $9.780 \text{ m/s}^2$ while the value at the poles is about $9.832 \text{ m/s}^2$!
**Satellites**

- Satellites have very high tangential speeds. The force of gravity from earth is supplying the centripetal acceleration necessary to keep them in orbit.

- Since satellites are significant distances from earth, it is no longer appropriate to use $g$ to calculate centripetal acceleration. Newton’s Universal Law of Gravitation is required.

- A **geosynchronous satellite** is one that always remains above the same point on the earth at the equator. In other words, this is a satellite with a period of rotation of 24 hours.
Conclusions

• This is just a taste of roughly the first third of Physics 7A at Berkeley.

• We began by understanding how physicists have strategized to quantify the world and break dimensions up into measurable units.

• We then used these units to describe motion in the study of **kinematics**, motion in the absence of forces.
  
  – We established the basic relationships between position, velocity, and acceleration along one direction.
  
  – Vectors and linear algebra are the mathematical tools that we developed to decompose complicated vectors along a set of simpler components, allowing us to extend these relationships to multiple dimensions.

• Then we studied **dynamics**, the effect of forces on an object’s motion.
  
  – Newton’s three laws of motion describe the relationship between forces, masses, and acceleration, allowing us to describe a variety of new situations that affect an object’s motion.
  
  – We then described a particular kind of motion, uniform circular motion, which is maintained by a constant centripetal force.
  
  – The source of this centripetal force in planetary motion is gravitational attraction between objects and we described its nature.

• We finished up by opening the door to a completely new way of analyzing motion with the abstractions of **work** and **energy**. These abstractions generalize the effect of forces on objects into potential and kinetic energy and can make analysis easier!

• Good luck with your engineering careers! There is so much more to come that will change the way you view the world around you.