WHAT IS FUZZY LOGIC AND WHAT DOES IT HAVE TO OFFER?

Lotfi A. Zadeh

Computer Science Division
Department of EECS
UC Berkeley

The Knight Lab
Helen Wills Neuroscience Institute
UC Berkeley
May 5, 2008

URL: http://www-bisc.cs.berkeley.edu
URL: http://www.cs.berkeley.edu/~zadeh/
Email: Zadeh@eecs.berkeley.edu

Research supported in part by ONR N00014-02-1-0294, BT Grant CT1080028046, Omron Grant, Tekes Grant, Chevron Texaco Grant and the BISC Program of UC Berkeley.
Science deals not with reality but with models of reality. In large measure, scientific progress is driven by a quest for better models of reality.
Informally,
\[ \text{fuzziness} = \text{unsharpness of class boundaries} \]

In the real world, fuzziness is a pervasive phenomenon.

To construct better models of reality it’s necessary to develop a better understanding of how to deal precisely with unsharpness of class boundaries. In large measure, fuzzy logic is motivated by this need.
WHAT IS FUZZY LOGIC?

- There are many misconceptions about fuzzy logic.
  - Fuzzy logic is fuzzy. wrong
  - Fuzzy logic is not fuzzy. Fuzzy logic is a precise logic of imprecision. correct
  - Fuzzy logic, FL, competes with probability theory, PT. wrong
  - Probability theory, PT, may be enriched through addition to PT of concepts and techniques drawn from fuzzy logic, FL. correct
PROBABILITY THEORY & FUZZY LOGIC?

wrong

FL

PT

PT+

correct

PT

PNL

PNL: precisiated natural language

6 /78

LAZ 5/2/2008
FUZZY LOGIC—AN OBJECT OF CONTROVERSY

- “Fuzzy theory is wrong, wrong, and pernicious. I can not think of any problem that could not be solved better by ordinary logic.” (Professor William Kahan, 1975)

- “Pattern recognition analysis and fuzzy logic analysis of breath VOCs independently distinguished healthy controls from hospitalized patients with 100% sensitivity and 100% specificity.” (Dr. Michael Phillips, 2007)

- “Because of this sophisticated functionality and performance realized by fuzzy logic, the number of sale of our Omron's blood pressure meter in all over the world is over 8,000,000 per year now.” (Hiroshi Nakajima, 2007)
FUZZY LOGIC—PRINCIPAL RATIONALES

Rationale A: precise modeling of imprecision

Rationale B: exploiting tolerance for imprecision through deliberate sacrifice of precision followed by precise modeling of imprecision. (Fuzzy Logic Gambit)
Clarification

- Imperfect information = information which is imprecise and/or uncertain and/or incomplete and/or partially true.

- Rationale A: fuzzy logic is designed to deal with ip-systems.

- Rationale B: if a p-system is associated with a tolerance for imprecision, then the tolerance for imprecision may be exploited through deliberate imprecisiation which reduces cost. Imprecisiation is followed by precisiation of meaning (fuzzy logic gambit).
Part 1: Early history

Part 2: What is fuzzy logic and what does fuzzy logic have to offer?
THE CONCEPT OF A FUZZY SET

- In the real world, most classes have unsharp boundaries and transition from membership to nonmembership is gradual rather than abrupt. Informally, a fuzzy set is a class with unsharp boundaries. A fuzzy set is associated with a membership function which assigns to each object its grade of membership in the fuzzy set.
EXAMPLE

- Vera is middle-aged \( \rightarrow \) Age(Vera) is middle-aged.
- Age of Vera is middle-aged \( \rightarrow \) precisiation

\[
\begin{array}{c}
\text{middle-age} \\
\text{definitely not middle-age} \\
\text{definitely middle-age} \\
\text{definitely not middle-age}
\end{array}
\]

\[\mu\]

\[
\begin{array}{c}
0 \quad 35 \quad 40 \quad 50 \quad 55
\end{array}
\]

\[
\begin{array}{c}
\text{possibility distribution}
\end{array}
\]

Familiar examples of fuzzy sets are: fuzzy set of tall men, fuzzy set of young women, fuzzy set of persons with long hair, fuzzy set of oval objects, etc.

In medicine, most concepts are fuzzy, that is are a matter of degree.

- Arthritis
- Hypertension
- Atherosclerosis
- Obesity
- ...
Basically, a natural language is a system for describing perceptions. Perceptions are intrinsically imprecise (fuzzy), reflecting the bounded ability of human sensory organs, and ultimately the brain, to resolve detail and store information. Imprecision of perceptions is passed on to natural languages.
SEMANTIC IMPRECISION (EXPLICIT)

EXAMPLES

WORDS/CONCEPTS
- Recession
- Slow
- Near
- Honesty
- Arthritis
- High blood pressure
- Civil war
- Cluster

PROPOSITIONS
- It is likely to be warm tomorrow.
- A box contains about 20 balls of various sizes. Most are small. There are many more small balls than large balls.
CONTINUED
EXAMPLES

PROPOSITIONS

- Usually most UA flights leave on time. Rarely most are delayed.
- It is very unlikely that there will be a significant decrease in the price of oil in the near future.

COMMANDS

- Slow down
- Slow down if foggy
- Park the car
SEMANTIC IMPRECISION (IMPLICIT)

EXAMPLES

- **Speed limit is 65 mph**
- **Checkout time is 1 pm**
NECESSITY OF IMPRECISION

- Can you explain to me the meaning of “Speed limit is 65 mph?”
- No imprecise numbers and no probabilities are allowed
- Imprecise numbers are allowed. No probabilities are allowed.
- Imprecise numbers are allowed. Precise probabilities are allowed.
- Imprecise numbers are allowed. Imprecise probabilities are allowed.
NECESSITY OF IMPRECISION

- Can you precisiate the meaning of “arthritis”?
- Can you precisiate the meaning of “recession”?
- Can you precisiate the meaning of “beyond reasonable doubt”?
- Can you precisiate the meaning of “causality”?

Precisiation of natural language is one of the principal contributions of fuzzy logic.
Early History
The concept of a fuzzy set is a simple, almost obvious concept. There is a compelling need for this concept for construction of better models of reality. It is this need that motivated my 1965 paper on fuzzy sets.

When I wrote my first paper on fuzzy sets my expectation was that the concept of a fuzzy set would gain rapid acceptance. This is not what happened. What I encountered was skepticism, derision and hostility.
• Among my few early supporters was my best friend, Richard Bellman, the father of dynamic programming. This is what he wrote when I sent him the manuscript of my paper, “Fuzzy Sets.”
Dear Lotfi:

I think that the paper is extremely interesting and I would like to publish it in JMAA, if agreeable to you. When I return, or while in Paris, I will write a companion paper on optimal decomposition of a set into subsets along the lines of our discussion.

Cordially,

Richard Bellman
Many others were not so kind. Here is a sample. Following the presentation of my first paper on the concept of a linguistic variable, Professor Rudolf Kalman, a brilliant scientist and a good friend of mine, had this to say:
“I would like to comment briefly on Professor Zadeh’s presentation. His proposals could be severely, ferociously, even brutally criticized from a technical point of view. This would be out of place here. But a blunt question remains: Is Professor Zadeh presenting important ideas or is he indulging in wishful thinking? No doubt Professor Zadeh’s enthusiasm for fuzziness has been reinforced by the prevailing climate in the U.S.—one of unprecedented permissiveness. ‘Fuzzification’ is a kind of scientific permissiveness; it tends to result in socially appealing slogans unaccompanied by the discipline of hard scientific work and patient observation.
In a similar vein, my esteemed colleague Professor William Kahan—a man with a brilliant mind—offered this assessment in 1975.

“Fuzzy theory is wrong, wrong, and pernicious.” says William Kahan, a professor of computer sciences and mathematics at Cal whose Evans Hall Office is a few doors from Zadeh’s. “I can not think of any problem that could not be solved better by ordinary logic.”
A country in which fuzzy set theory and fuzzy logic were welcomed with open arms was Japan. Just a year after the publication of my first paper on fuzzy sets, I received two letters, one of which follows.
Dear Prof. Zadeh:

My name is Hidemitsu Ogawa, I am the official engineer of the Ministry of International Trade and Industry, Japanese Government. I am making research in pattern recognition at the Electrotechnical Laboratory.

I read with very interest your paper on “Fuzzy Sets” in Information and Control. I will tell you my impression. I think that the concept of a fuzzy set is very important for problems of pattern recognition and information processing. In these fields, the notion of a fuzzy set provides us with good visibility.”

Sincerely,

Hidemitsu Ogawa
Electrotechnical Laboratory
Important contributions to fuzzy set theory began to appear in Japanese journals in 1968. The early pioneers were Professors Asai, Tanaka and Terano. Here is a sample of early papers.


"Fuzzy logic is the prodigal technology of the 90’s. Invented by a University of California, Berkeley, professor in 1964, it has found far greater understanding and acceptance in Japan, where it is used in everything from subways to washing machines.”
“Now, amid ever sharper concerns over America’s technological slippage—counterposed by the Clinton administration’s high-tech zeal—fuzzy logic is finally finding some respect. Of all the sciences intended to make machines behave more like humans—including artificial intelligence, virtual reality, and neural networks—fuzzy logic appears to have the inside track in practical applications.”
WHAT IS FUZZY LOGIC
and
WHAT DOES FUZZY LOGIC HAVE TO OFFER?
Basically, fuzzy logic is a precise system of concepts and techniques aimed at formalization/mechanization of two remarkable human capabilities (a) to converse, reason and make rational decisions in an environment of imprecision, uncertainty, incompleteness of information and partiality of truth; and (b) the capability to perform a wide variety of physical and mental tasks (e.g., driving a car in city traffic, without any measurements and any computations).
The cornerstones of fuzzy logic are graduation, granulation, precisiation and granular constraint.

One of the most important features of fuzzy logic is its high power of precisiation.

(cointensive mm-precisiation)
In bivalent logic, BL, truth is bivalent, implying that every proposition, \( p \), is either true or false, with no degrees of truth allowed.

In multivalent logic, ML, truth is a matter of degree.

In fuzzy logic, FL:
- everything is, or is allowed to be, to be graduated, i.e., a matter of degree
- everything is, or is allowed to be, granulated (linguistic)
Fuzzy logic is much more general than traditional logical systems. The greater generality of fuzzy logic is needed to deal with complex problems in the realms of search, question-answering decision and control. Fuzzy logic provides a foundation for the development of new tools for dealing with natural languages and knowledge representation.
WHAT IS FUZZY LOGIC?

fuzzy logic (FL) is aimed at a formalization of modes of reasoning which are approximate rather than exact examples:

exact

- all men are mortal
- Socrates is a man
- Socrates is mortal

approximate

- most Swedes are tall
- Magnus is a Swede
- it is likely that Magnus is tall
CONTINUED

fuzzy logic (FL) has four principal facets

$FL/L \leftarrow \text{logical }$ (narrow sense FL)

$FL/E \rightarrow \text{epistemic}$

$FL/R \leftarrow \text{relational}$

$FL/S \leftarrow \text{set-theoretic}$

$F: \text{fuzziness/ fuzzification}$

$G: \text{granularity/ granulation}$

$F.G: F \text{ and } G$
ACCEPANCE OF FUZZY LOGIC

In the evolution of science a time comes when alongside the brilliant successes of a theory, T, what becomes visible are classes of problems which fall beyond the reach of T. At that point, the stage is set for a progression from T to T*--a generalization of T.

Among the many historical examples are the transitions from Newtonian mechanics to quantum mechanics; from linear system theory to nonlinear system theory; and from deterministic models to probabilistic models in economics and decision analysis.

Fuzzy logic is a better approximation to reality
In this perspective, a fundamental point—a point which is not as yet widely recognized—is that there are many classes of problems which cannot be addressed by any theory, $T$, which is based on bivalent logic. The problem with bivalent logic is that it is in fundamental conflict with reality—a reality in which almost everything is a matter of degree. To deal with reality what is needed is fuzzy logic.
THE TRIP-PLANNING PROBLEM

- I have to fly from A to D, and would like to get there as soon as possible
- I have two choices: (a) fly to D with a connection in B; or (b) fly to D with a connection in C

- if I choose (a), I will arrive in D at time $t_1$
- if I choose (b), I will arrive in D at time $t_2$
- $t_1$ is earlier than $t_2$
- therefore, I should choose (a)?
Continued

- now, let us take a closer look at the problem
- the connection time, $c_B$, in B is short
- should I miss the connecting flight from B to D, the next flight will bring me to D at $t_3$
- $t_3$ is later than $t_2$
- what should I do?

$$\text{decision} = f\left(t_1, t_2, t_3, c_B, c_C\right)$$

existing methods of decision analysis do not have the capability to compute $f$

reason: nominal values of decision variables ≠ observed values of decision variables
what we are beginning to appreciate—and what Lord Kelvin did not—is the fundamental importance of the remarkable human capability to perform a wide variety of physical and mental tasks without any measurements and any computations.

in performing such tasks, exemplified by driving a car in city traffic, we employ perceptions of distance, speed, time, position, shape, likelihood, intent, similarity and other attributes of physical and mental objects.
**MEASUREMENT-BASED VS. PERCEPTION-BASED INFORMATION**

**INFORMATION**

- **measurement-based**
  - numerical
  - *it is 35 °C*
  - *Eva is 28*

- **perception-based**
  - linguistic
  - *It is very warm*
  - *Eva is young*
  - *it is cloudy*
  - *traffic is heavy*
  - *it is hard to find parking near the campus*

- *measurement-based information may be viewed as special case of perception-based information*
Dana is young

Tandy is a few years older than Dana

Tandy is ?A

Y is several times larger than X

Y is large

X is ?A

small X + small Y = medium

medium X + large Y = large

X is ?A, Y is ?B
simple examples

Dana is young
Tandy is a few years older than Dana
Tandy is (young + few)

most Swedes are tall
most Swedes are blond
(\overset{2}{\overset{\text{most}}{\overset{1}{\text{most}}}}) Swedes are tall and blond
most Swedes are tall
most Swedes are not tall
There is a deep-seated tradition in science of striving for the ultimate in rigor and precision.

Words are less precise than numbers.

Why and where, then, should words be used?

1. When the available information is perception-based or not precise enough to justify the use of numbers.

2. When there is a tolerance for imprecision which can be exploited to achieve tractability, simplicity, robustness and low solution cost.

3. When the expressive power of words is greater than the expressive power of numbers.
one of the most basic concepts in science is that of a variable

- numerical (X=5; X=(3, 2); …)
- linguistic (X is small; (X, Y) is much larger)

a linguistic variable is a variable whose values are words or sentences in a natural or synthetic language (Zadeh 1973)

the concept of a linguistic variable plays a central role in fuzzy logic and underlies most of its applications
example: Age
primary terms: young, middle-aged, old
modifiers: not, very, quite, rather, ...
linguistic values: young, very young, not very young and not very old, ...

\[
\begin{align*}
\mu & \quad \text{young} & \quad \text{middle-aged} & \quad \text{old} \\
0 & \quad 1 & \quad 1 & \quad 0 \\
\end{align*}
\]
EXAMPLES OF F-GRANULATION (LINGUISTIC VARIABLES)

color: red, blue, green, yellow, ...

age: young, middle-aged, old, very old

size: small, big, very big, ...

distance: near, far, very, not very far, ...

• humans have a remarkable capability to perform a wide variety of physical and mental tasks, e.g., driving a car in city traffic, without any measurements and any computations

• one of the principal aims of CTP is to develop a better understanding of how this capability can be added to machines
PRINCIPAL APPLICATIONS OF FUZZY LOGIC

- control
- consumer products
- industrial systems
- automotive
- decision analysis
- medicine
- geology
- pattern recognition
- diagnostics

CFR: calculus of fuzzy rules
CALCULUS
OF FUZZY
RULES
CALCULUS OF FUZZY RULES (CFR)

- **syntax**: legal forms of rules
  - if X is A then Y is B
  - if X is A then Y is B unless Z is C

- **taxonomy**: classification of rules
  - **categorical**
    - if X is then Y is B
  - **qualified**
    - if X is A then usually (Y is B)

- **semantics**: meaning of rules
  - **single rule**
  - **collection of rules**
DEPENDENCY AND COMMAND

● Dependency
  Y is large if X is small
  Y is medium if X is medium
  Y is small if X is large

● Command
  reduce Y slightly if X is small
  reduce Y substantially if X is not small
FUZZY IF-THEN RULES

- increase interest rates slightly if unemployment is low and inflation is moderate
- increase interest rates sharply if unemployment is low and inflation is moderate but rising sharply
- decrease interest rates slightly if unemployment is low but increasing and inflation rate is low and stable
GRANULATION OF A DYNAMICAL SYSTEM
YAMAKAWA’S INVERTED PENDULUM (1989)
MATHEMATICAL MODEL (\(\nu\)-PRECISE)

\[
I \ddot{\theta} = VL \sin \theta - HL \cos \theta,
\]

\[
\dot{V} - mg = -mL(\dot{\theta} \sin \theta + \dot{\theta}^2 \cos \theta),
\]

\[
H = m\ddot{y} + mL(\ddot{\theta} \cos \theta - \dot{\theta}^2 \sin \theta),
\]

\[
U - H = M\ddot{y},
\]
PERCEPTION-BASED (LINGUISTIC MODEL)
v-IMPRECISE, mm-PRECISE

\[
\begin{align*}
& \text{IF } \theta \text{ is PM and } \dot{\theta} \text{ is ZR, then } \dot{y} \text{ is PM,} \\
& \quad \text{also} \\
& \text{IF } \theta \text{ is PS and } \dot{\theta} \text{ is PS, then } \dot{y} \text{ is PS,} \\
& \quad \text{also} \\
& \text{IF } \theta \text{ is PS and } \dot{\theta} \text{ is NS, then } \dot{y} \text{ is ZR,} \\
& \quad \text{also} \\
& \text{IF } \theta \text{ is NM and } \dot{\theta} \text{ is ZR, then } \dot{y} \text{ is NM,} \\
& \quad \text{also} \\
& \text{IF } \theta \text{ is NS and } \dot{\theta} \text{ is NS, then } \dot{y} \text{ is NS,} \\
& \quad \text{also} \\
& \text{IF } \theta \text{ is NS and } \dot{\theta} \text{ is PS, then } \dot{y} \text{ is ZR,} \\
& \quad \text{also} \\
& \text{IF } \theta \text{ is ZR and } \dot{\theta} \text{ is ZR, then } \dot{y} \text{ is ZR.}
\end{align*}
\]
Control Rules:

1. If (speed is low) and (shift is high) then (-3)
2. If (speed is high) and (shift is low) then (+3)
3. If (throt is low) and (speed is high) then (+3)
4. If (throt is low) and (speed is low) then (+1)
5. If (throt is high) and (speed is high) then (-1)
6. If (throt is high) and (speed is low) then (-3)
INTERPOLATION

\[ Y \text{ is } B_1 \text{ if } X \text{ is } A_1 \]
\[ Y \text{ is } B_2 \text{ if } X \text{ is } A_2 \]

\[ \ldots \ldots \ldots \]
\[ Y \text{ is } B_n \text{ if } X \text{ is } A_n \]
\[ Y \text{ is } ?B \text{ if } X \text{ is } A \]

\[ A \neq A_1, \ldots, A_n \]

Conjunctive approach (Zadeh 1973)

NEW TOOLS

Computing with numbers:
- CN
- IA
- PT

Computing with intervals:
- PNL

Computing with words:
- CTP
- PFT
- UTU
- THD
- PTp

CTP: Computational theory of perceptions
PFT: Protoform theory
PTp: Perception-based probability theory
THD: Theory of hierarchical definability
UTU: Unified Theory of uncertainty
THE BASICS OF PNL

- The point of departure in PNL is the key idea:
  - A proposition, p, drawn from a natural language, NL, is precisiated by expressing its meaning as a generalized constraint

\[ p \rightarrow X \text{ isr } R \]

- In general, X, R, r are implicit in p
- precisiation of p \[ \rightarrow \] explicitation of X, R, r

constraining relation
Identifier of modality (type of constraint)
constrained (focal) variable
SIMPLE EXAMPLE

- Eva is young  Age(Eva) is young

- Annotated representation
  \[ X / \text{Age}(Eva) \quad \text{is} \quad R / \text{young} \]
KEY POINTS

- A proposition is an answer to a question

  example:
  
  \( p: \text{Eva is young} \)

  is an answer to the question
  
  \( q: \text{How old is Eva?} \)

- The concept of a generalized constraint serves as a basis for generalized-constraint-based semantics of natural languages
Representing the meaning of a proposition as a generalized constraint reduces the problem of computation with information described in natural language to the problem of computation with generalized constraints. In large measure, computation with generalized constraints involves the use of rules which govern propagation and counterpropagation of generalized constraints. Among such rules, the principal rule is the deduction principle (Zadeh 1965, 1975).
EXTENSION PRINCIPLE (POSSIBILISTIC)

- \( X \) is a variable which takes values in \( U \), and \( f \) is a function from \( U \) to \( V \). The point of departure is a possibilistic constraint on \( f(X) \) expressed as \( f(X) \) is \( A \) where \( A \) is a fuzzy relation in \( V \) which is defined by its membership function \( \mu_A(v), \quad v \in V \).

- \( g \) is a function from \( U \) to \( W \). The possibilistic constraint on \( f(X) \) induces a possibilistic constraint on \( g(X) \) which may be expressed as \( g(X) \) is \( B \) where \( B \) is a fuzzy relation. The question is: What is \( B \)?
CONTINUED

\[ f(X) \text{ is } A \frac{\text{subject to}}{g(X) \text{ is } B} \]

\[ \mu_B(w) = \sup_u \mu_A(f(u)) \]

subject to

\[ w = g(u) \]

\(\mu_A\) and \(\mu_B\) are the membership functions of \(A\) and \(B\), respectively.
 STRUCTURE OF THE EXTENSION PRINCIPLE

\[ f^{-1}(A) \quad \mu_A(f(u)) \]

counterpropagation

\[ f^{-1}(A) \quad g(f^{-1}(A)) \]

propagation

U

V

W
CONCLUSION

- Existing scientific theories are based on bivalent logic—a logic in which everything is black or white, with no shades of gray allowed.
- What is not recognized, to the extent that it should, is that bivalent logic is in fundamental conflict with reality.
- Fuzzy logic is not in conflict with bivalent logic—it is a generalization of bivalent logic in which everything is, or is allowed to be, a matter of degree.
- Fuzzy logic provides a foundation for the methodology of computing with words and perceptions.
RELATED PAPERS


RELATED PAPERS BY L.A.Z IN REVERSE CHRONOLOGICAL ORDER

From computing with numbers to computing with words -- from manipulation of measurements to manipulation of perceptions, IEEE Transactions on Circuits and Systems 45, 105-119, 1999.


CONTINUED


Factual Information About the Impact of Fuzzy Logic

PATENTS

- Number of fuzzy-logic-related patents applied for in Japan: 17,740
- Number of fuzzy-logic-related patents issued in Japan: 4,801
- Number of fuzzy-logic-related patents issued in the US: around 1,700

Number of papers in INSPEC and MathSciNet which have "fuzzy" in title:

<table>
<thead>
<tr>
<th>Period</th>
<th>INSPEC - &quot;fuzzy&quot; in title</th>
<th>MathSciNet - &quot;fuzzy&quot; in title</th>
</tr>
</thead>
<tbody>
<tr>
<td>1970-1979</td>
<td>569</td>
<td>443</td>
</tr>
<tr>
<td>1980-1989</td>
<td>2,403</td>
<td>2,465</td>
</tr>
<tr>
<td>1990-1999</td>
<td>23,214</td>
<td>5,487</td>
</tr>
<tr>
<td>2000-present</td>
<td>24,910</td>
<td>6,217</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>51,096</strong></td>
<td><strong>14,612</strong></td>
</tr>
</tbody>
</table>
JOURNALS  ("fuzzy" in title)
1. Fuzzy in title
2. Fuzzy Sets and Systems
3. IEEE Transactions on Fuzzy Systems
4. Fuzzy Optimization and Decision Making
5. Journal of Intelligent & Fuzzy Systems
6. Fuzzy Economic Review
10. International Review of Fuzzy Mathematics
11. Fuzzy Systems and Soft Computing