To facilitate understanding of the basic concepts which underlie precisiation of meaning, a clarification dialogue is included in the Appendix.
INTRODUCTION
WHAT IS INFORMATION?

- Our age is the age of information.
- Information has a position of centrality in modern society.
- So what is information?
- 64 years ago Shannon suggested in his path-breaking work that information is what reduces uncertainty.
- Shannon’s work was the beginning of the information age.
CONTINUED

- Shannon’s information theory did not address a basic issue—meaning of information. Minsky and Bar-hilel wrote a paper on semantic information theory. Their paper did not resolve the issue of meaning of information.

- Understanding of meaning is a prerequisite to use of information as a basis for decision-making. But understanding of meaning is not sufficient.
What is needed for computation with information described in a natural language goes beyond understanding. More concretely, what is needed is precisiation of meaning. A system of computation which offers a capability to precisiate and compute with information described in natural language is Computing with Words (CW or CWW).
CONTINUED

- CW has two modules: Precisiation module and Computation module. In what follows, the focus of attention is Precisiation.
Precisiation of meaning will be discussed in greater detail at a later point. In the following, discussion is informal.

Basically, precisiation involves construction of a computational model.

Simple examples:
Most Swedes are tall

Proportion(tall. Swedes/Swedes) is most

\[ \mu \]

height 160cm 180cm 200cm

\[ \mu \]

0 0.5 1

Proportion

CONTINUED
The concept of precisiation of meaning suggests a simple way of clarifying the distinction between Computing with Words and Natural Language Processing.

- In Computing with Words, the objects of computation are precisiated words.
- In Natural Language Processing, the objects of computation are unprecisiated words.
The coming decade is likely to be a decade of automation of everyday reasoning and decision-making. In the world of automated reasoning and decision-making, computation with natural language is certain to play a prominent role.
CONTINUED

- A prerequisite to computation is precisiation of meaning. As was noted earlier, precisiation of meaning lies beyond understanding of meaning.
MEANING VS. PRECISIATION OF MEANING—EXAMPLES

Robert: Keep under refrigeration.
Lotfi: I understand what you mean, but could you precisiate your meaning of “Keep under refrigeration?”

Robert: Vera is middle-aged
Lotfi: I understand what you mean, but could you precisiate your meaning of “middle-aged?”
Natural languages are intrinsically imprecise. Basically, a natural language is a system for describing perceptions. Perceptions are imprecise, reflecting the bounded ability of human sensory organs and ultimately the brain, to resolve detail and store information. Imprecision of perceptions is passed on to natural languages.
NATURAL LANGUAGE AND PERCEPTIONS

- $p$: perception
- $NL(p)$: description of $p$; semantic entity
- $p^+$: perceptions evoked by $NL(p)$
- $p^+$: meaning of $p$; denotation of $p$
There are many different forms of imprecision in natural languages. A principal source of imprecision is unsharpness of class boundaries.

Everyday examples:

Words (phrases, predicates)
- tall
- near
- not very tall
- mountain
- hand
- high fever
- several large balls
- recession
CONTINUED

Propositions

- Most Swedes are tall
- Icy roads are slippery
- Speed limit is 65 mph
- Check out time is 1pm

Commands

- Keep under refrigeration
- Handle with care
UNSHARPNESS OF CLASS BOUNDARIES = FUZZINESS

- In natural language, words and phrases are—in large measure—labels of classes with unsharp (fuzzy) boundaries.

- Fuzziness of words is a concomitant of fuzziness of perceptions.

- Fuzziness of natural languages is rooted in unsharpness of class boundaries.

- Fuzzy set = precisiated (graduated) class with unsharp boundaries.
Graduation/precisiation = association of a class which has unsharp boundaries with a scale of degrees—more concretely, with a membership function. Degrees are allowed to be fuzzy (fuzzy sets of type 2).
It is important to differentiate between two forms of graduation: (a) Attribute-based (intensional); and (b) Name-based (extensional). Graduation is attribute-based if it assigns to an object, a, a grade of membership which is a function of attributes of a. Graduation is name-based if no attributes are involved.
REPRESENTATION OF DEGREES AND VALUES

- Degree or value
  - Numerical
  - Linguistic
  - Fuzzy
  - Visual

- 0.85
- 0.0
- 1.0
- High

- Z-mouse
- F-mark

LAZ 9/22/2010
Humans have a remarkable capability to graduate perceptions without any measurements or any computations. More specifically, assume that I am given an object, a, and a class, A, and am asked to put a mark on a scale from 0 to 1 indicating my perception of the degree to which a fits A. Generally, I would have no difficulty in doing this, even when no measurable attributes are involved.
EXAMPLES

- To what degree do you like Chinese cuisine?
- To what degree do you prefer Chinese cuisine to Indian cuisine?
- To what degree do you like your job?
- What is the probability that Obama will be reelected?
I believe that Robert is very honest
EXAMPLE—GRADUATION/PRECISIATION OF MIDDLE-AGE

- Imprecision of meaning = fuzziness of meaning
- Computational model of middle-age (trapezoidal fuzzy set)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>μ</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>40</td>
<td>0</td>
</tr>
<tr>
<td>40</td>
<td>45</td>
<td>0.8</td>
</tr>
<tr>
<td>45</td>
<td>55</td>
<td>1</td>
</tr>
<tr>
<td>55</td>
<td>60</td>
<td>0</td>
</tr>
</tbody>
</table>

*Note: Parameters are context-dependent*
Assume that Vera is 43 years old.

The statement “Vera’s grade of membership in middle-age is 0.8,” may be interpreted as “the truth-value of the proposition “Vera is middle-age” given that she is 43, is 0.8. An equivalent interpretation is: Given that Vera is middle-aged, the possibility that she is 43 is 0.8.
HMC—HONDA FUZZY LOGIC TRANSMISSION

Control Rules:
1. If (speed is low) and (shift is high) then (-3)
2. If (speed is high) and (shift is low) then (+3)
3. If (throt is low) and (speed is high) then (+3)
4. If (throt is low) and (speed is low) then (+1)
KEY POINT—REPRESENTATION OF FUZZY DEGREES

- degree
- perception of degree (fuzzy)
- z-mouse
- word
- membership function

f-mark

0
1

high

0
1

LAZ 9/22/2010
A Z-mouse is an electronic implementation of a spray pen. The cursor is a round fuzzy mark called an f-mark. The color of the mark is a matter of choice. A dot identifies the centroid of the mark. The cross-section of an f-mark is a trapezoidal fuzzy set with adjustable parameters.
MORE ON Z-MOUSE

- If I am not sure what the degree is, and I am allowed to use a Z-mouse, I will put a fuzzy f-mark on the scale.
- A fuzzy f-mark reflects imprecision of my perception.
- A Z-mouse reads my f-mark and represents it internally into a trapezoidal fuzzy set—a fuzzy set which serves as an object of computation for the machinery of Computing with Words.
In most cases, a crisp mark should be interpreted as the centroid of a fuzzy mark. For example, if I am asked to estimate the probability that Obama will be able to solve the financial crisis, and I put a crisp mark at .7, the crisp mark should be interpreted as the centroid of my fuzzy perception of the probability that Obama will be able to solve the financial crisis.
CONTINUED

- What this points to is that more often than not crisp probabilities are in reality centroids of fuzzy probabilities.

- Most real-world probabilities are fuzzy probabilities. In large measure, this reality is not reflected in traditional theories of probability.
EXAMPLE OF APPLICATION OF Z-MOUSE

- I am scheduled to fly from San Francisco to Los Angeles. My flight is scheduled to leave at 5pm. I have to be at the airport about an hour before departure. Usually it takes about forty five minutes to get to the airport from my home. I would like to be pretty sure that I arrive at the airport in time. At what time should I leave my home?
Time of departure

- Usually:
  - Time of departure from home
    - 5pm
    - 4pm

Time of arrival

- Specified/computed:
  - 3:50pm

f-mark

- Usually:
  - 1

Time of departure from home

- Specified:
  - .8

Trial

- Specified:
  - 0

LAZ 9/22/2010
A Z-mouse serves primarily as a means of visual fuzzy data entry and retrieval. Computation of an answer to a question is carried out through the use of the machinery of Computing with Words (CW or CWW).

Precisiation of meaning is a prerequisite to computation with information described in natural language.
BASIC STRUCTURE OF PRECISIATION

precisiiend \rightarrow \text{precisiation} \rightarrow \text{precisiand}

\text{precisiation language}

\text{p: object of precisiation}
\text{p*: result of precisiation}

\text{cointension}

- \text{precisiand} = \text{model of meaning}
- \text{extension} = \text{name-based meaning}
- \text{intension} = \text{attribute-based meaning}
- \text{cointension} = \text{qualitative measure of proximity of meanings} = \text{qualitative measure of proximity of the model (precisiand) and the object of modeling (precisiend)}
A precisiiend has a multiplicity of precisiiands.

Generally, achievement of cointensive precisiation requires that if the precisiiend is fuzzy so must be the precisiiand.

Crisp definitions of fuzzy concepts is the norm in science. What is widely unrecognized is that in general crisp definitions of fuzzy concepts are not cointensive.
**PHASES OF CW**

\[
CW = \text{[PRECISIATION $\rightarrow$ COMPUTATION]}
\]

- **Phase 1**
  - Precisiation
  - Precisiation module
  - \( q \)
  - \( I \)

- **Phase 2**
  - Computation
  - Computation module
  - \( q^* \)
  - \( I^* \)

- **Granular computing**
- **Ans\((q/I)\)**

- **Fuzzy logic**

- Precisiation and computation employ the machinery of fuzzy logic.
What is meant by graduation of propositions? If I were asked to graduate the proposition, \( p \): Most Swedes are tall, what would I do? What is the connection between graduation of \( p \) and precisiation of \( p \)?
In general, a proposition, \( p \), may be associated with a variety of attributes. A basic attribute is the truth-value of \( p \), \( t(p) \). In this perspective, graduation of \( p \) may be related to graduation of truth-value of \( p \). As will be seen later, graduation of truth-value of \( p \) is a byproduct of precisiation of \( p \). However, what should be noted is that not every proposition has a truth-value.
The truth-value of $p$ cannot be assessed in isolation. If I were asked what is the truth-value of $p$: Most Swedes are tall, I would have to know how most and tall are defined, and be given the distribution of heights of Swedes. Let us call the needed knowledge the Information Base, $IB(p)$. 
The question is: How can the truth-value be computed given the information base, $IB(p)$? What is needed for this purpose is generalized-constraint-based semantics, GCS. Generalized-constraint-based semantics is rooted in test-score semantics (Zadeh 1981, 1986).
Generalized-constraint-based semantics is a generalization of truth-conditional and possible-world semantics. In the following, precisiation of propositions through the use of generalized-constraint-based semantics is discussed in greater detail.
PRECISIATION OF MEANING— GENERALIZED CONSTRAINT-BASED SEMANTICS
The concept of a proposition is one of the most basic concepts in the realms of both natural and synthetic languages. A dictionary definition of a proposition reads: an expression in language or signs of something that can be believed, doubted, denied or is either true or false. Simple examples:

- Robert is very bright
- Leslie is much taller than Ixel
- Most Swedes are tall.
CONTINUED

- In CW, traditional definitions of the concept of a proposition are put aside. What is employed is a definition which lends itself to computation.
As was noted earlier, precisiation of meaning is a prerequisite to computation with information described in a natural language.

The issue of precisiation of propositions has a position of centrality in CW.

Precisiation of propositions drawn from a natural language is—in large measure—beyond the reach of traditional approaches to semantics of natural languages.
Let $p$ be a proposition drawn from a natural language.

$p$ is a carrier of information.

Information is a constraint (restriction) on the values which a variable is allowed to take.

As a carrier of information, $p$ may be viewed as an expression of an underlying constraint.
However, standard constraints are not sufficiently general to serve this purpose.

A standard constraint on a variable, $X$, is expressed as:

$$X \in R$$

where $R$ is a crisp relation. Standard constraints have no elasticity.

To achieve generality, what is needed is the concept of a generalized constraint.
A generalized constraint, GC(X), is an expression of the form:

\[ X \text{ isr } R \]

where \( X \) is the constrained variable, \( R \) is the constraining relation and \( r \) is an indexical variable which defines the modality of the constraint, that is, the way in which \( R \) constrains \( X \). Typically, \( R \) is a fuzzy relation.
KEY IDEA

- In GCS, $p$ is represented/precised as a generalized constraint—a constraint which plays the role of a computational model of $p$
R is a restriction on the values of X.

Typically, R is a fuzzy set.

R plays the role of a predicate.

r defines the way in which R constrains X.
When \( p \) is viewed as a generalized constraint, three basic questions arise.

The answers to these questions define the meaning of \( p \).
1. What is the constrained variable, $X$, in $p$?
2. What is the constraining relation, $R$, in $p$?
3. How does $R$ constrain $X$?

- Typically, $X$, $R$ and $r$ are implicit in $p$ rather than explicit in $p$. 
STANDARD VS. GENERALIZED CONSTRAINTS

Examples:

- **Standard (crisp) constraint:**
  \[ 2 \leq X \leq 5 \]
  Standard constraints have no elasticity.

- **Generalized constraint:**
  Usually \( X \) is larger than approximately 2 and smaller than approximately 5.
  Generalized constraints have elasticity.
Elasticity is needed for representation of meaning of propositions drawn from a natural language.

Propositions drawn from a natural language have elasticity in the sense that their meaning can be stretched.

Example: Vera is middle-aged
A key idea which underlies precisiation of meaning in CW—an idea which differentiates precisiation of meaning in CW from traditional approaches to representation of meaning in natural languages—is that of representing the computational model of p as a generalized constraint.
In general, in the case of propositions drawn from a natural language, the constrained variable and the constraining relation are implicit rather than explicit.
The concept of a generalized constraint bridges the divide between linguistics and mathematics.
There are many levels of complexity in precisiation, depending on how easy or difficult it is to represent a proposition, $p$, as a generalized constraint. There are two principal levels of complexity: Level 1 (precisiation is basic); and Level 2 (precisiation is advanced).
Informally, precisiation is Level 1 (basic) if $X$ and $R$ are explicit or nearly explicit. Precisiation is Level 2 (advanced) if $X$ and/or $R$ are implicit.

Typically, in a proposition drawn from a natural language $X$ and $R$ are implicit. For such propositions—referred to as implicit propositions—advanced precisiation is needed.
Typically, if $p$ is a proposition drawn from a mathematical language, $X$ and $R$ are explicit. For such propositions—referred to as explicit propositions—advanced precisiation is not needed.
LEVELS OF COMPLEXITY IN PRECISIATION

Precisiation

- **Level 1 (basic)**
  - X and R are explicit or nearly explicit
  - Pressure is low
  - Robert is tall

- **Level 2 (advanced)**
  - X and/or R are implicit
  - Most Swedes are tall
  - Swedes are much taller than Italians
In parallel with levels of complexity in precisiation, there are levels of complexity in CW. In Level 1 (basic CW), precisiation is Level 1 (basic). In Level 2 (advanced CW), precisiation is Level 2 (advanced).

Both conceptually and computationally, basic CW is much simpler than advanced CW.
IMPORTANT NOTE

● In propositions drawn from the language of linguistic variables and fuzzy if-then rules, X and R are explicit, meaning that such propositions are explicit.

● At this juncture, most of the applications of CW involve basic CW, centered on the calculi of linguistic variables and fuzzy if-then rules.
Basic CW is associated with an extensive literature. For this reason, in the following attention is focused on advanced CW.

In coming years, there will be a gradual shift from basic CW to advanced CW.
In most applications, especially in the realm of natural languages, only three primary constraints and their combinations are employed.

Primary constraints: possibilistic \((r=\text{blank})\); probabilistic \((r=p)\); and veristic \((r=v)\). Preponderantly, the constraints are possibilistic.
EXAMPLES: POSSIBILISTIC

- Robert is tall $\rightarrow$ Height(Robert) is tall
  $\uparrow$
  $\uparrow$
  X
  $\uparrow$
  R
  blank

- Most Swedes are tall $\rightarrow$
  Count(tall.Swedes/Swedes) is most
  $\uparrow$
  $\uparrow$
  X
  $\uparrow$
  R
  blank
EXAMPLES: PROBABILISTIC

- $X$ is a normally distributed random variable with mean $m$ and variance $\sigma^2$ \[ X \sim N(m, \sigma^2) \]

- $X$ is a random variable taking the values $u_1, u_2, u_3$ with probabilities $p_1, p_2$ and $p_3$, respectively \[ X \sim (p_1\|u_1+p_2\|u_2+p_3\|u_3) \]
EXAMPLES: VERISTIC

- Robert is half German, quarter French and quarter Italian
  
  Ethnicity (Robert) isv (0.5|German + 0.25|French + 0.25|Italian)

- Robert resided in London from 1985 to 1990
  
  Reside (Robert, London) isv [1985, 1990]
In representation of $p$ as a generalized constraint, $p: X \in S R$, there are two important points that have to be noted. First, $X$ need not be a scalar variable. $X$ may be vector-valued or, more generally, have the structure of a semantic network.
For example, in the case of the proposition, $p$: Robert gave a ring to Anne, $X$ may be represented as the 3-tuple $(\text{Giver}, \text{Recipient}, \text{Object})$, with the corresponding values of $R$ being $(\text{Robert}, \text{Anne}, \text{Ring})$. 
Second, in general, X is not unique. However, it is usually the case that among possible choices either there is one that has higher plausibility than others, or there are a few that are closely related. For example, if

\[ p: \text{Leslie is much taller than Ixel} \]

then a plausible choice of X is

\[ X: \text{Height(Leslie)} \]
in which case the corresponding constraining relation is

\[ R: \text{Much taller than Ixel.} \]

Another plausible choice is

\[ X: (\text{Height(Leslie)}; \text{Height(Ixel)}). \]

Correspondingly,

\[ R: \text{Much taller} \]
With regard to the third question, the constraint in the proposition: Robert is tall, is possibilistic in the sense that it defines possible values of Height (Robert), with the understanding that possibility is a matter of degree.
When the meaning of $p$ is represented as a generalized constraint, the expression $X$ isr $R$ is referred to as the canonical form of $p$, CF($p$). Thus,

$$CF(p): X \text{ isr } R$$

The concept of a canonical form of $p$ plays an important role in precisiation of meaning of $p$. 
It is important to note that the use of generalized constraints in precisiation of propositions drawn from a natural language is greatly facilitated by the fact that, as noted earlier, natural language constraints are for the most part possibilistic—and hence are easy to manipulate.
The concept of explanatory database (ED)

- In generalized-constraint-based semantics, the concept of an explanatory database, ED, serves as a basis for precisiation of meaning of p. (Zadeh 1984) More concretely, ED is a collection of relations, with the names of relations drawn, but not exclusively, from the constituents of p.
Basically, ED may be viewed as the information which is needed to define $X$ and $R$. For example, for the proposition, $p$: Most Swedes are tall, ED may be represented as:

$$ED = \text{POPULATION.SWEDES}[\text{Name};\text{Height}]+\text{TALL}[\text{Height};\mu]+\text{MOST}[\text{Proportion};\mu],$$

where + plays the role of comma.
**DATABASE VARIABLES**

- Database variables are entries in relations in ED.
- **Example:**
  
  Database variables in the ED of: Most Swedes are tall, are: $\text{Height}(\text{Name}_1), \ldots, \text{Height}(\text{Name}_n)$, $\mu_{\text{tall}}$ and $\mu_{\text{most}}$, where $\mu_{\text{tall}}$ and $\mu_{\text{most}}$ are the membership functions of tall and most, respectively.

- **ED is instantiated when the values of database variables are specified.** Instantiated ED is denoted as $ED^+$. 
It is important to note that definition of X and R may be viewed as precisiation of X and R. Precisiation of X and R is needed because X and R are described in a natural language.

In relation to possible world semantics, $ED^+$ may be viewed as the description of a possible world.
ADDITIONAL EXAMPLE OF ED

p: Brian is much taller than most of his friends.

X: Height of Brian.

R: Much taller than most of his friends.

ED = HEIGHT [Name; Height] + FRIENDS.BRIAN [Name; µ] + MUCH.TALLER [Height1; Height2; µ] + MOST [Proportion; µ]
In FRIENDS.BRIAN, $\mu$ is the degree to which Name is a friend of Brian.
It is important to note that relations in ED are uninstantiated, that is, the values of database variables, $v_1, ..., v_m$—entries in relations in ED—are not specified.
THE CONCEPT OF A PRECISIATED CANONICAL FORM, CF*(p)

- After X and R have been identified and the explanatory database, ED, has been constructed, X and R may be defined as functions of ED, that is, functions of database variables. As was noted earlier, definition of X and R may be viewed as precisiation of X and R. Precisiated X and R are denoted as X* and R*, respectively.
A canonical form, $\text{CF}^*(p)$, with precisiated values of $X$ and $R$, $X^*$ and $R^*$, is referred to as a precisiated canonical form, and is expressed as:

$$C^*(p): X^* \text{ isr } R^*$$

The concept of a precisiated canonical form has a position of centrality in GCS.
FROM $p$ TO $\text{CF}^* (p)$:

$X^* \text{ isr } R^*$
The concepts discussed so far provide a basis for a relatively straightforward procedure for constructing the precisiated canonical form of a given proposition, $p$. The precisiated canonical form, together with the procedure which leads to it, constitutes the computational model of $p$. In CW, the objects of computation are computational models of propositions.
In possible world semantics and, more generally, in logic, the meaning of $p$ is equated to its intension, that is, a mapping from possible worlds to truth-values. The concept of a precisiated canonical form may be viewed as a generalization of the concept of intension. To underscore this connection, a precisiated canonical form of $p$ may be referred to as a generalized intension, denoted as $GI(p)$. 
A summary of the procedure for computing the precisiated canonical form of $p$ is presented in the following.
A preliminary step is that of clarification, if needed, of the meaning of the given proposition. This step requires world knowledge.

Examples:

- Overeating causes obesity
- Most of those who overeat are obese.

- Obesity is caused by overeating
- Most of those who are obese, overeat.
CONTINUED

- Young men like young women
  Most young men like mostly young women.
- Swedes are much taller than Italians
  Most Swedes are much taller than most Italians.
  The average height of Swedes is much greater than the average height of Italians.
CONTINUED

- Clarification of a predicate.

Most tall Swedes

clarification¹  Mostly tall Swedes

clarification²  Most of tall Swedes
PROCEDURE

- Step 1. Identification (explicitation) of X and R.
  Identify the constrained variable, X, and the corresponding constraining relation, R. R depends on X. Generally, X and R are identified by inspection.
Step 2. Construction of ED.

What information is needed to precisiate (define) X and R? An answer to this question identifies the explanatory database, ED. Equivalently, ED may be viewed as an answer to the question: What information is needed to compute the truth-value of p?
Step 3. Precisiation of $X$ and $R$.

How can the information in ED be used to precisiate the values of $X$ and $R$? This step leads to precisiated values of $X$ and $R$, $X^*$ and $R^*$, and thus results in the precisiated canonical form, $CF^*(p)$.

Precisiated $X^*$ and $R^*$ may be expressed as functions of ED and, more specifically, as functions of database variables, $v_1, \ldots, v_m$. 
More concretely, $X^*$ and $R^*$ may be expressed as:

$$X^* = f_1(v_1, \ldots, v_m)$$

$$R^* = f_2(v_1, \ldots, v_m)$$
When $X^*$ and $R^*$ are expressed as functions of database variables, $v_1, \ldots, v_m$, the precisiated canonical form induces a generalized constraint on database variables, expressed as $GC(V)$, $V=(v_1, \ldots, v_m)$. $GC(V)$ may be interpreted as the possibility distribution of database variables, given $p$. The possibility distribution induced by $p$ may be viewed as the generalized intension of $p$, $GI(p)$. 
It is important to observe that in the case of possibilistic constraints $CF^{*}(p)$ induces a possibilistic constraint, $GC(V)$, on database variables, $v_1, \ldots, v_m$. More concretely, $GC(V)$ may be expressed as:

$$GC(V): f(v_1, \ldots, v_m) \text{ is } A,$$

where $A$ is a fuzzy set in the space of database variables.
TRUTH-VALUE OF p

- The truth-value of p depends on instantiated ED, ED+. The truth-value of p, t(p, ED+), may be computed by assessing the degree to which the generalized constraint, X* isr R*, or, equivalently GC(V), is satisfied. It is important to observe that the possibility of an instantiated ED given p is equal to the truth-value of p given instantiated ED (Zadeh 1981).

- End of procedure.
NOTE

- It should be noted that not every proposition has a truth-value. Example: I believe that Robert is honest.
SUMMARY OF PROCEDURE

Identification of $X$ and $R$

Step 1

$X, R$

Explanatory database

Step 2

$ED$

Precisiated canonical form

Step 3

$X^* isr R^*$

Generalized intension

$GI(p)$

Computational model of $p$
CONCLUSION

- In GCS, a precisiated meaning of a proposition, $p$, is equated to its precisiated canonical form, $X^* \text{ isr } R^*$, with the understanding that $X^* \text{ isr } R^*$ is described as a procedure.

- The precisiated canonical form translates into a generalized constraint on database variables, $\text{GC}(V)$, $V=v_1, \ldots, v_m$. $\text{GC}(V)$ plays the role of a computational model of $p$. 

EXAMPLE

- Note. In the following example \( r = \text{blank}, \) that is, the generalized constraints are possibilistic.

1. \( p: \) Most Swedes are tall

Clarification. Clarification not needed

Step 1. Identification (explicitation) of \( X \) and \( R. \)

\( X \) is identified as the proportion of tall Swedes among Swedes.
Correspondingly, $R$ is identified as Most.

Digression.

In fuzzy logic, proportion is defined as a relative $\Sigma \text{Count}$. (Zadeh 1983) More specifically, if $A$ and $B$ are fuzzy sets in $U$, $U=\{u_1, \ldots, u_n\}$, the $\Sigma \text{Count}(\text{cardinality})$ of $A$ is defined as:

$$\Sigma \text{Count}(A) = \Sigma_{i} \mu_{A}(u_i)$$
The relative $\Sigma\text{Count}$ of $B$ in $A$ is defined as:

$$\Sigma\text{Count}(B / A) = \frac{\Sigma\text{Count}(A \cap B)}{\Sigma\text{Count}(A)}$$

$$= \frac{\sum_i (\mu_A(u_i) \land \mu_B(u_i))}{\sum_i \mu_A(u_i)}$$

where $\cap =$intersection and $\land =$min
In application to the example under consideration, assume that the height of \( i \)th Swede, \( \text{Name}_i \), is \( h_i \) and that the grade of membership of \( h_i \) in tall is \( \mu_{\text{tall}}(h_i) \), \( i=1, \ldots, n \). \( X \) may be expressed as:

\[
X = \frac{1}{n} \left( \sum_i \mu_{\text{tall}}(h_i) \right)
\]

Step 2. Construction of ED. The information which is needed to define \( X \) and \( R \) is contained in the explanatory database, \( ED \), where
Continued

\[ ED = \text{POPULATION.SWEDES[Name; Height]} + \]
\[ \text{TALL[Height; } \mu \text{]} + \]
\[ \text{MOST[Proportion; } \mu \text{]} \]

Step 3. Precisiation of X and R.
In relation to ED, precisiated X and R may be expressed as:

\[ X^* = \frac{1}{n} \left( \sum_i \mu_{\text{tall}}(h_i) \right) \]

\[ R^* = \text{MOST[Proportion; } \mu \text{]} \]
The precisiated canonical form is expressed as:

$$CF^*p = X^* \text{ is } R^*$$

where

$$X^* = \frac{1}{n} \left( \sum_i \mu_{tall}(h_i) \right)$$

$$R^* = \text{MOST}[\text{Proportion}; \mu]$$
The truth-value of $p$, $t(p, ED)$, is the degree to which the constraint $X^* \text{ isr } R^*$ is satisfied. More concretely,

$$t(p, ED^+) = \mu_{\text{most}} \left( \frac{1}{n} \sum_{i} \mu_{\text{tall}}(h_i) \right)$$

Note. The right-hand side of this equation may be viewed as a constraint on database variables $h_1, \ldots, h_n$, $\mu_{\text{tall}}$ and $\mu_{\text{most}}$. 
SUMMATION

● Natural languages are pervasively imprecise, especially in the realm of meaning. The primary source of imprecision is unsharpness of class boundaries. In this sense, words, phrases, propositions and commands in natural languages are preponderantly imprecise.

● Precisiation of meaning is a prerequisite to computation.
Precisiation of meaning is a prerequisite to achievement of higher levels of mechanization of natural language understanding. Precisiation of meaning plays a particularly important role in Computing with Words and communication between humans and machines. Humans can understand imprecise language but machines cannot.
Despite its intrinsic importance, precisiation of meaning has drawn little, if any, attention within linguistics and computational linguistics. There is a reason. In large measure, theories of natural languages are based on bivalent logic.
Bivalent logic is not the right logic for dealing with imprecision of natural languages. What is needed for this purpose is fuzzy logic. Basically, fuzzy logic is the logic of classes with unsharp boundaries.
In the approach presented in this lecture, the point of departure is an unconventional definition of the concept of a proposition.
More specifically, $p$ is interpreted as a constraint. In general, the constrained variable, $X$, and the constraining relation, $R$, are implicit in $p$. Basically, $R$ restricts the values which $X$ is allowed to take. Usually, $R$ is a fuzzy set. Since there are many ways in which $R$ can constrain $X$, the constraint on $X$ is referred to as a generalized constraint.
A generalized constraint is an expression of the form, $X isr R$, in which $X$ is the constrained variable, $R$ is the constraining relation and $r$ is an indexical variable which defines the way in which $R$ constrains $X$. The primary constraints are possibilistic ($r=$blank); probabilistic ($r=p$); and veristic ($r=v$).
In GCS, a precisiated meaning of p is represented as a generalized constraint on database variables. The generalized constraint on database variables plays the role of a computational model of p. Generally, the constraint is possibilistic or a combination of a possibilistic constraint and a probabilistic constraint. Usually, X and R can be identified by inspection.
CONTINUED—CONCLUSION

- Representation of $p$ as a generalized constraint is the centerpiece of generalized-constraint-based semantics, GCS. Through precisiation of meaning, GCS opens the door to a wide-ranging enlargement of the role of natural languages in scientific theories.

- The concept of a generalized constraint bridges the divide between linguistics and mathematics.
Precisiation of meaning plays a pivotal role in CW. What should be underscored, however, is that in Level 1 (basic) precisiation, precisiation of meaning does not require the elaborate machinery which is described in this section. In Level 1 (basic) precisiation, constraints are explicit or nearly explicit.
An important case in point is the language of linguistic variables and fuzzy if-then rules. For this language, advanced precisiation is not needed. The language of linguistic variables and fuzzy if-then rules plays a pivotal role in most practical applications of fuzzy logic. In part, this is a reflection of the fact that for the language of linguistic variables and fuzzy if-then rules, basic precisiation is sufficient.
APPENDIX
EXAMPLE

Swedes are much taller than Italians.

Clarification.

Most Swedes are much taller than most Italians.

Step 1. Identification of X and R.

X = Proportion of Swedes who are much taller than most Italians.

R = Most
Step 2. Construction of ED

POPULATION.SWEDES[Name; Height] +
POPULATION.ITALIANS[Name; Height] +
MUCH.TALLER[Height1; Height2; μ] +
MOST[Proportion; μ]
Step 3. Precisiation of X and R

Precisiation of X

Find the height of ith Swede, NameS_i, i=1, ..., m

\[ h_i = \text{Height} \text{POPULATION.SWEDES}[\text{Name}= \text{NameS}_i; \text{Height}] \]

Find the height of jth Italian, NameI_j, j=1, ..., n

\[ k_j = \text{Height} \text{POPULATION.ITALIANS}[\text{Name}= \text{NameI}_j; \text{Height}] \]
EXAMPLE

Find the degree, $a_{ij}$, to which NameS$_i$ is much taller than Name$_j$

$$a_{ij} = \mu \text{MUCH.TALLER}[\text{Height}_1 = h_i; \text{Height}_2 = k_j; \mu]$$

Compute the proportion, $p_i$, of Italians in the relation to whom NameS$_i$ is much taller

$$p_i = \frac{1}{n} \sum_j a_{ij}$$
CONTINUED

Compute the degree, $q_i$, to which $p_i$ satisfies most

$$q_i = MOST \left[ \frac{1}{n} \sum_j a_{ij}; \mu \right]$$

Compute the proportion, $X^*$, of Swedes who are much taller than most Italians

$$X^* = \frac{1}{m} \sum_i q_i$$
Precisiation of $R$

$R^* = \text{MOST}[\text{Proportion}; \mu]$

The precisiated canonical form may be expressed as $\text{CF}^*(p) = X^* \text{ is } R^*$, where

$$X^* = \frac{1}{m} \sum_i q_i$$

$R^* = \text{MOST}[\text{Proportion}; \mu]$

Additional examples may be found in cited papers.
The basic ideas which underlie precisiation of meaning and, more particularly, generalized-constraint-based semantics, are actually quite simple. To bring this out, it is expedient to supplement a formal exposition of GCS with an informal narrative in the form of a dialogue between Robert and Lotfi. In large measure, the narrative is self-contained.
Robert: Lotfi, generalized-constraint-based semantics looks complicated to me. Can you explain in simple terms the basic ideas which underlie GCS?
Lotfi: I will be pleased to do so. Let us start with an example, $p$: Most Swedes are tall. $p$ is a proposition. As a proposition, $p$ is a carrier of information. Without loss of generality, we can assume that $p$ is a carrier of information about a variable, $X$, which is implicit in $p$. If I asked you what is this variable, what would you say?
Robert: As I see it, p tells me something about the proportion of tall Swedes among Swedes.

Lotfi: Right. What does p tell you about the value of the variable?

Robert: To me, the value is not sharply defined. I would say it is fuzzy.

Lotfi: So what is it?

Robert: It is the word “most.”
Lotfi: You are right. So what we see is that $p$ may be interpreted as the assignment of a value “most” to the variable, $X$: Proportion of tall Swedes among Swedes.
As you can see, a basic difference between a proposition drawn from a natural language and a proposition drawn from a mathematical language is that in the latter the variables and the values assigned to them are explicit, whereas in the former the variables and the assigned values are implicit.
There is an additional difference. When $p$ is drawn from a natural language, the assigned value is not sharply defined—typically it is fuzzy, as “most” is. When $p$ is drawn from a mathematical language, the assigned value is sharply defined.

Robert: I get the idea. So what comes next?
Lotfi: There is another important point. When p is drawn from a natural language, the value assigned to X is not really a value of X—it is a constraint (restriction) on the values which X is allowed to take. This suggests an unconventional definition of a proposition, p, drawn from a natural language. Specifically, a proposition is an implicit constraint on an implicit variable.
I should like to add that the constraints which I have in mind are not standard constraints—they are so-called generalized constraints.
Robert: What is a generalized constraint? Why do we need generalized constraints?

Lotfi: A generalized constraint is expressed as:

\[ X \text{ isr } R \]
where $X$ is the constrained variable, $R$ is the constraining relation—typically a fuzzy set—and $r$ is an indexical variable which defines how $R$ constrains $X$. Let me explain why the concept of a generalized constraint is needed in precisiation of meaning of a proposition drawn from a natural language.
Standard constraints are hard in the sense that they have no elasticity. In a natural language, meaning can be stretched. What this implies is that to represent meaning, a constraint must have elasticity. To deal with richness of meaning, elasticity is necessary but not sufficient. Consider the proposition: Usually most flights leave on time.
What is the constrained variable and what is the constraining relation in this proposition? Actually, for most propositions drawn from a natural language a large repertoire of constraints is not necessary. What is sufficient are three so-called primary constraints and their combinations. The primary constraints are: possibilistic, probabilistic and veristic.
Here are simple examples of primary constraints:

- **Possibilistic constraint:**
  Robert is possibly French and possibly German

- **Probabilistic constraint:**
  With probability 0.75 Robert is German
  With probability 0.25 Robert is French

- **Veristic constraint:**
  Robert is three-quarters German and one-quarter French
The role of primary constraints is analogous to the role of primary colors: red, green and blue. In most cases, constraints are possibilistic. Possibilistic constraints are much easier to manipulate than probabilistic constraints.
Robert: Could you clarify what you have in mind when you talk about elasticity of meaning?

Lotfi: I admit that I did not say enough. Let me elaborate. In a natural language, meaning can be stretched. Consider a simple example, Robert is young. Assume that young is a fuzzy set and Robert is 30.
Furthermore, assume that in a particular context the grade of membership of 30 in young is 0.8. To apply young to Robert, the meaning of young must be stretched. To what degree? In fuzzy logic, the degree of stretch is equated to \((1 - \text{grade of membership of 30 in young})\). Thus, the degree of stretch is 0.2.
Furthermore, the grade of membership of 30 in young is interpreted as the possibility that Robert is 30, given that Robert is young. What this implies is that the fuzzy set young defines the possibility distribution of the variable Age (Robert). Note that the fuzzy set young is a restriction on the values which the variable Age (Robert) can take.
It is in this sense that the proposition Robert is young is a possibilistic constraint on Age (Robert).

Now, in a natural language almost all words and phrases are labels of fuzzy sets. What this means is that in a natural language the meaning of words and phrases can be stretched, as in the Robert example.
It is in this sense that words and phrases in a natural language have elasticity. Another important point. What I have said so far explains why in the realm of natural languages most constraints are possibilistic. This is equivalent to saying what I said already, namely, that in a natural language most words and phrases are labels of fuzzy sets.
Robert: Many thanks. You clarified what was not clear to me.
Lotfi: May I add that there is an analogy that may be of assistance. More specifically, the fuzzy set young may be represented as a chain linked to a spring, as shown in the next viewgraph. The left end of the chain is fixed and the position of the right end of the spring represents the value of the variable Age (Robert).
The force that is applied to the right end of the spring is a measure of grade of membership. Initially, the length of the chain is 0, as is the length of the spring.
Robert: Many thanks for the explanation. The analogy helps to understand what you mean by elasticity of meaning.

Lotfi: I should like to add that elasticity of meaning is a basic characteristic of natural languages. Elasticity of meaning is a neglected issue in the literatures of linguistics, computational linguistics and philosophy of languages. There is a reason.
Traditional theories of natural language are based on bivalent logic. Bivalent logic, by itself or in combination with probability theory, is not the right tool for dealing with elasticity of meaning. What is needed for this purpose is fuzzy logic. In fuzzy logic everything is or is allowed to be a matter of degree.
Robert: Thanks again for the clarification. Going back to where we left off suppose I figured out what is the constrained variable, $X$, and the constraining relation, $R$. Is there something else that has to be done?
Lotfi: Yes, there is. You see, X and R are described in a natural language. What this means is that we are not through with precisiation of meaning of p. What remains to be done is precisiation (definition) of X and R.
For this purpose, we construct a so-called explanatory database, ED, which consists of a collection of relations in terms of which X and R can be defined. The entries in relations in ED are referred to as database variables. Unless stated to the contrary, database variables are assumed to be uninstantiated.
Robert: Can you be more specific?
Lotfi: To construct ED you ask yourself the question: What information—in the form of a collection or relations—is needed to precisiate (define) X and R? Looking at p, we see that to precisiate X we need two relations: POPULATION.SWEDES[Name; Height] and TALL[Height; μ].
In the relation TALL[Height; \(\mu\)], \(\mu\) is the grade of membership of a value of Height, \(h\), in the fuzzy set tall. So far as \(R\) is concerned, the needed relation is MOST[Proportion; \(\mu\)], where \(\mu\) is the grade of membership of a value of Proportion in the fuzzy set Most.
CONTINUED

Equivalently, it is frequently helpful to ask the question: What is the information which is needed to assess the degree to which p is true?
At this point, we can express ED as the collection:

\[ ED = \text{POPULATION.SWEDES}[\text{Name; Height}] + \text{TALL}[\text{Height; } \mu] + \text{MOST}[\text{Proportion; } \mu] \]

in which for convenience plus is used in place of comma.
Robert: So, we have constructed ED for the proposition, \( p \): Most Swedes are tall. More generally, given a proposition, \( p \), how difficult is it to construct ED for \( p \)?

Lotfi: For humans it is easy. A few examples suffice to learn how to construct ED. Construction of ED is easy for humans because humans have world knowledge. At this juncture, we do not have an algorithm for constructing ED.
Robert: Now that we have ED, what comes next?

Lotfi: We can use ED to precisiate (define) X and R. Let us start with X. In words, X is described as the proportion of tall Swedes among Swedes. Let us assume that in the relation POPULATION.SWEDES there are \( n \) names. Then the proportion of tall Swedes among Swedes would be the number of tall Swedes divided by \( n \).
Here we come to a problem. Tall Swedes is a fuzzy subset of Swedes. The question is: What is the number of elements in a fuzzy set? In fuzzy logic, there are different ways of answering this question. The simplest is referred to as the $\Sigma$Count. More concretely, if $A$ is a fuzzy set with a membership function $\mu_A$, then the $\Sigma$Count of $A$ is defined as the sum of grades of membership in $A$. 

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In application to the number of tall Swedes, the \( \Sigma \)Count of tall Swedes may be expressed as:

\[
\Sigma \text{Count(tall.Swedes)} = \sum_{i=1}^{n} \mu_{\text{tall}}(h_i)
\]

where \( h_i \) is the height of \( \text{Name}_i \).

Consequently, the proportion of tall Swedes among Swedes may be written as:

\[
X = \frac{1}{n} \left( \sum_{i=1}^{n} \mu_{\text{tall}}(h_i) \right)
\]
This expression may be viewed as a precisiation (definition) of \( X \) in terms of \( ED \). More specifically, \( X \) is expressed as a function of database variables \( h_1, \ldots, h_n, \mu_{\text{tall}} \) and \( \mu_{\text{most}} \).

Precisiation (definition) of \( R \) is simpler. Specifically, \( R = \text{Most} \), where Most is a fuzzy set. At this point, we have precisiated (defined) \( X \) and \( R \) in terms of \( ED \).
Robert: So what have we accomplished?

Lotfi: We started with a proposition, \( p \): 
Most Swedes are tall. We interpreted \( p \) as a generalized (possibilistic) constraint. We identified the constrained variable, \( X \), as the proportion of tall Swedes among Swedes. We identified the constraining relation, \( R \), as a fuzzy set, Most. Next, we constructed an explanatory database, \( ED \).
Finally, we precisiated (defined) $X$, $R$ and $q$ in terms of $ED$, that is, as function of database variables $h_1, \ldots, h_n$, $\mu_{\text{tall}}$ and $\mu_{\text{most}}$. In this way, we precisiated the meaning of $p$, which was our objective. The precisiated meaning may be expressed as the constraint:

$$\frac{1}{n} \left( \sum_{i=1}^{n} \mu_{\text{tall}}(h_i) \right) \text{ is Most}$$

Robert: So, you precisiated the meaning of $p$. What purpose does it serve?
CONTINUED

Lotfi: The principal purpose is the following. Unprecisiated (raw) propositions drawn from a natural language cannot be computed with. Precisiation is a prerequisite to computation. What is important to understand is that precisiation of meaning opens the door to computation with natural language.
Robert: Sounds great. I am impressed. However, it is not completely clear to me what you have in mind when you say “opens the door to computation with natural language.” Can you clarify it?

Lotfi: With pleasure. Computation with natural language or, more or less equivalently, Computing with Words (CW or CWW), is largely unrelated to natural language processing.
More specifically, computation with natural language is focused on computation with information described in a natural language. Typically, what is involved is solution of a problem which is stated in a natural language. Let me go back to our example, $p$: Most Swedes are tall. Given this information, how can you compute the average height of Swedes?
Robert: Frankly, your question makes no sense to me. Are you serious? How can you expect me to compute the average height of Swedes from the information that most Swedes are tall?

Lotfi: That is conventional wisdom. A mathematician would say that the problem is ill-posed. It appears to be ill-posed for two reasons.
CONTINUED

First, because the given information: Most Swedes are tall, is fuzzy, and second, because you assume that I am expecting you to come up with a crisp answer like “the average height of Swedes is 5’ 10.” Actually, what I expect is a fuzzy answer—it would be unreasonable to expect a crisp answer.

Robert: Thanks for the clarification. I am beginning to see the point of your question.
CONTINUED

Lotfi: I should like to add a key point. The problem becomes well-posed if $p$ is precisiated. This is the essence of Computing with Words.
Robert: I am beginning to understand the need for precisiation, but my understanding is not complete as yet. Can you explain how the average height of Swedes can be computed from precisiated p?

Lotfi: Recall that precisiated p is a possibilistic constraint expressed as:

\[ \frac{1}{n} \left( \sum_{i=1}^{n} \mu_{tall}(h_i) \right) \] is Most
From the definition of a possibilistic constraint it follows that the constraint on $X$ may be rewritten as:

$$t = \mu_{\text{most}} \left( \frac{1}{n} \sum_{i=1}^{n} \mu_{\text{tall}}(h_i) \right)$$

What this expression means is that given the $h_i$, $\mu_{\text{tall}}$ and $\mu_{\text{most}}$, we can compute the degree, $t$, to which the constraint is satisfied.
It is this degree, \( t \), that is the truth-value of \( p \). Now, here is a key idea. The precisiated \( p \) constrains \( X \). \( X \) is a function of database variables. It follows that ultimately what \( p \) constrains are database variables. This has important implications. Let me elaborate.
What we see is that the constraint induced by $p$ on the $h_i$ is of the general form

$$f(h_1, \ldots, h_n)$$

is Most

What we are interested in is the induced constraint on the average height of Swedes. The average height of Swedes may be expressed as:

$$h_{\text{ave}} = \frac{1}{n} \left( \sum_{i=1}^{n} h_i \right)$$
This expression is of the general form
\[ g(h_1, \ldots, h_n) \text{ is } \tilde{h}_{\text{ave}} \]
where \( \tilde{h}_{\text{ave}} \) is a fuzzy set that we want to compute.
At this stage, we can employ the Extension Principle of fuzzy logic to compute $h_{\text{ave}}$ (Zadeh 1975 I, II & III). In general terms, this principle tells us that from a given possibilistic constraint of the form

$$f(x_1, \ldots, x_n) \text{ is } A$$

in which $A$ is a fuzzy set, we can derive an induced possibilistic constraint on $g(x_1, \ldots, x_n)$,

$$g(x_1, \ldots, x_n) \text{ is } ?B,$$
CONTINUED

in which $B$ is a fuzzy set defined by the solution of the mathematical program

$$\mu_B(v) = \sup_{x_1, \ldots, x_n} \mu_A(f(x_1, \ldots, x_n))$$

subject to

$$v = g(x_1, \ldots, x_n)$$

In application to our example, what we see is that we have reduced computation of the average height of Swedes to the solution of the mathematical program
\[ \mu_B(v) = \sup_{h_1, \ldots, h_n} \mu_{\text{most}}(f(h_1, \ldots, h_n)) \]

subject to

\[ v = \frac{1}{n} \left( \sum_{i=1}^{n} h_i \right) \]

In effect, this is the solution to the problem which I posed to you. As you can see, reduction of the original problem to the solution of a mathematical program is not so simple.
CONTINUED

However, solution of the mathematical program to which the original problem is reduced, is well within the capabilities of desktop computers.
Robert: I am beginning to see the basic idea. Through precisiation, you have reduced the problem of computation with information described in a natural language—a seemingly ill-posed problem—to a well-posed tractable problem in mathematical programming. I am impressed by what you have accomplished, though I must say that the reduction is nontrivial.
Without your explanation, it would be hard to see the basic ideas. I can also see why computation with natural language is a move into a new and largely unexplored territory. Thank you for clarifying the import of your statement: precisiation of meaning opens the door to computation with natural language.
Lotfi: I appreciate your comment. May I add that I believe—but have not verified it as yet—that in closed form the solution to the mathematical program may be expressed as:

\[ h_{\text{ave}} \geq \text{Most} \times \text{Tall} \]

where \( \text{Most} \times \text{Tall} \) is the product of fuzzy numbers Most and Tall.

Robert: This is a very interesting result, if true. It agrees with my intuition.
Lotfi: I appreciate your comment. I would like to conclude our dialogue with a prediction. As we move further into the age of machine intelligence and automated reasoning, the complex of problems related to computation with information described in a natural language, is certain to grow in visibility and importance.
The informal dialogue between Robert and Lotfi has come to an end.