In one form or another, attempts to construct a logic of everyday reasoning go back to antiquity. The classical, Aristotelian, bivalent logic may be viewed as a product of such attempts. However, Aristotelian logic does not qualify as a logic of everyday reasoning because it does not come to grips with a core issue—the intrinsic imprecision of everyday reasoning.
In modern times, logical systems—driven by a quest for the ultimate in precision, rigor and depth—have become increasingly estranged from everyday reasoning. But times are changing. The coming decade is likely to be a decade of automation of everyday reasoning and decision-making.
In the world of automated everyday reasoning and decision-making, the logic of everyday reasoning is certain to play an important role.

More concretely, humans have many remarkable capabilities. One such capability is the capability to converse, communicate, reason and make rational decisions in an environment of imprecision, uncertainty, incompleteness of information and partiality of truth.
A related capability is the capability to identify and summarize decision-relevant information.

Another remarkable capability is the capability to perform a wide variety of physical and mental tasks—such as driving a car in city traffic—without any measurements and any computations.

Development of a logic of everyday reasoning is a prerequisite to automation (mechanization) of these and related capabilities.
At this juncture, a logic of everyday reasoning is not in existence. Intrinsic Imprecision of everyday reasoning is closely linked to intrinsic imprecision of natural languages. It is this imprecision that precludes the possibility of constructing a logic of everyday reasoning in the spirit of traditional logical systems. A departure is required.
In the following, two approaches to the construction of a logic of approximate reasoning are outlined. The first approach relates to what is called precisiable everyday reasoning. The second approach relates to unprecisiable everyday reasoning. In the following, a few simple examples will be used to illustrate the difference.
I hail a taxi and ask the driver to take me to address A. There are two versions: (a) I ask the driver to take me to A the shortest way; and (b) I ask the driver to take me to A the fastest way. Version (a) is precisiable. There exists a provably valid (p-valid) solution. Version (b) is unprecisiable. There does not exist a p-valid solution. What exists is a fuzzily valid (f-valid) solution.
Based on his/her experience, the driver chooses route (a) for (a) and route (b) for (b).

In Version (a), if there is a map of the area, it is possible to construct the shortest way to A. This would be a p-valid solution. Thus, for Version (a) there exists a p-valid solution but the driver’s choice of route (a) may be viewed as an f-valid solution which in some sense is good enough.
In Version (b), it is not possible to construct a mathematically well defined model of the system and hence it is not possible to construct a p-valid solution. The problem is rooted in uncertainties related to traffic conditions, timing of lights, etc. In fact, if the driver had asked me to define what I mean by “the fastest way,” I could not come up with an answer to his/her question. In conclusion, in Version (b) there does not exist a p-valid solution. What exists is an f-valid solution.
The assertion that there does not exist a p-valid solution in Version (b) requires a clarification. Given a problem, p, it is always possible to construct a model of p, Mp, which is simple enough to admit a p-valid solution. The problem is that Mp is not realistic. An unstated assumption is that a model of p should be realistic.
Informally, cointension of $p$ and $M_p$, $C(p, M_p)$, is a qualitative measure of the proximity—in some specified sense of proximity—of $p$ and $M_p$.

$M_p$ is a cointensive model of $p$ if $C(p, M_p)$ is high.
Let $p$ be a problem, subject of reasoning, proposition, a collection of propositions, a construct, etc.

Informally, $p$ is precisiable if there exists a cointensive model of $p$ which is mathematically well defined.

If $p$ is unprecisiable, then $p$ cannot be proved or disproved. $p$ may be f-valid.
EXAMPLE—CAUSALITY

- $p$: Unrest in Nigeria caused a three dollar-a-barrel increase in the price of oil.

- $p$ is unprecisiable, but it may be $f$-valid.

- Note. What is widely unrecognized is that many assertions regarding causality are unprecisiable.

- Frequently, it is incorrectly concluded that $A$ caused $B$, based on the observation that $B$ followed $A$. 
EXAMPLE

- \( p: \) Icy roads are slippery.

**Slippery roads are dangerous.**

**Icy roads are dangerous.**

- \( p \) is precisiable. Conclusion is incorrect.
CONSTRUCTION OF A COINTENSIVE MODEL OF P

- Icy roads are slippery → Most icy roads are slippery → $Q_1$A’s are B’s
- Slippery roads are dangerous → Most slippery roads are dangerous → $Q_2$B’s are C’s
- Icy roads are dangerous → ?$Q_3$A’s are C’s
INTERSECTION/PRODUCT SYLLOGISM

Q₁A’s are B’s
Q₂(A and B)’s are C’s
Q₁×Q₂ A’s are (B and C)’s

Special cases
all A’s are B’s
all B’s are C’s
all A’s are C’s

almost all A’s are B’s
almost all B’s are C’s
[0,1] A’s are C’s
• Almost all A’s are B’s
• Almost all B’s are C’s
• No A’s are C’s
Why does the reasoning appear to be correct?

Icy roads are slippery
Slippery roads are dangerous
Icy roads are dangerous

Because an implicit assumption is made.
Slippery roads are icy.
CHAINING SYLLOGISM (ZADEH 1986)

\[ Q_1 \text{A’s are B’s} \]
\[ Q_1 \text{B’s are A’s} \]
\[ Q_2 \text{B’s are C’s} \]
\[ (Q_1 + Q_2 - 1) \text{A’s are C’s} \]

Most A’s are B’s
Most B’s are A’s
Most B’s are C’s
\[ (2 \text{Most} - 1) \text{A’s are C’s} \]

● This conclusion agrees with intuition
EXAMPLE—A PRECISIABLE PROBLEM

- Usually Robert leaves office at about 5pm. Usually it takes Robert about an hour to get home from work. At what time does Robert get home?

- An answer in the spirit of everyday reasoning is: Usually Robert gets home at about 6pm. Is this answer correct?

- An answer based on precisiable logic of everyday reasoning is shown in the following.
SOLUTION

time of arrival = time of departure + travel time
X, Y and Z are random variables with respective probability densities $p_X$, $p_Y$, $p_Z$, respectively. All integrals are definite integrals.

\[ Z = X + Y \]

\[ p_Z = p_X \circ p_Y \text{ (composition)} \]

\[ p_Z(v) = \int p_X(u) p_Y(v - u) \, du \]
Generalized constraint on $p_X$

$$\int p_X(u) \mu_{*a}(u) du$$

is usually

*a stands for approximately 5pm.

Generalized constraint on $p_Y$

$$\int p_Y(u) \mu_{*b}(u) du$$

is usually

*b stands for approximately 1 hour.
Apply the Extension Principle (Zadeh 1965) to

\[
\int p_x(u) \mu_{*a}(u) du \quad \text{is usually}
\]

\[
\int p_y(u) \mu_{*b}(u) du \quad \text{is usually}
\]

\[
\int p_x(u) du = 1
\]

\[
\int p_y(u) du = 1
\]

\[
p_z = p_x \circ p_y
\]
\[ \mu_{p_Z}(v) = \sup_{p_X, p_Y}(\mu_{\text{usually}}(\int p_X(u)\mu_{*a}(u)du)) \land \\
\mu_{\text{usually}}(\int p_Y(u)\mu_{*b}(u)du) \]

subject to

\[ v = p_X \circ p_Y \]

\[ \int p_X(u)du = 1 \]

\[ \int p_Y(u)du = 1 \]
PRECISIABLE LOGIC OF EVERYDAY REASONING
Precisiable logic of everyday reasoning is based on Computing with Words (Zadeh 1996, 1999).
WHAT IS COMPUTING WITH WORDS (CW or CWW)?

- Computing with Words is a system of computation and deduction which offers an important capability that traditional systems do not have—a capability to compute with and infer from information described in natural language.
The point of departure in CW is a question, q, of the form: What is the value of a variable, X? Associated with q is a question-relevant information set, I, expressed as X is I, meaning that an answer to q, Ans(q/I), is to be deduced (computed) from I.
Typically, I consists of a collection of propositions, \( p_1, \ldots, p_n \), in which some or all of the \( p_i \), \( i=1, \ldots, n \), are expressed in a natural language. The \( p_i \) are carriers of information about X.
Furthermore, some of the \( p_i \) may be drawn from external sources of information, typically from world knowledge. A \( p_i \) drawn from an external source is identified as \( +p_i \).

Computation of \( \text{Ans}(q/l) \) is described in part by an aggregation function, \( f \).

Precisiation is a prerequisite to computation.
**PRECISIATION OF MEANING**

$p$ \(\xrightarrow{\text{precisiation}}\) \(p^*\)

- $p^*$ is intended to serve as an object of computation, that is, $p^*$ is computation-ready.
- $p^*$ is mathematically well-defined.

$\text{Vera is middle-aged}$

$\rightarrow$ $\text{Age(Vera) is middle-aged}$
CW= [PRECISIATION → COMPUTATION]

Phase 1
precisiation

precisiation module

Phase 2
computation

computation module

Granular computing

q

q*

I

I*

fuzzy logic

Ans(q/I)
**PRECISIATION VIA GRADUATION**

- Graduation of a fuzzy concept or a fuzzy set, $A$, serves as a means of precisiation of $A$.

**Examples**
- Graduation of middle-age
- Graduation of the concept of earthquake via the Richter Scale
- Graduation of recession?
- Graduation of mountain?
EXAMPLE—MIDDLE-AGE

- Imprecision of meaning = elasticity of meaning
- Elasticity of meaning = fuzziness of meaning
Control Rules:
1. If (speed is low) and (shift is high) then (-3)
2. If (speed is high) and (shift is low) then (+3)
3. If (throt is low) and (speed is high) then (+3)
4. If (throt is low) and (speed is low) then (+1)
Granulation serves as a means of imprecisiation (coarsening of information).
GRADUATION / GRANULATION

- graduation = precisiation
- granulation = imprecisiation
A granule in a universe of discourse, $U$, is a clump of elements of $U$ drawn together by indistinguishability, equivalence, similarity, proximity or functionality.

A granule is precisiated through association with a generalized constraint.
### Basic Concepts—Singular and Granular Values

**Universe of Discourse** $U$

- **Granular Value of $X$**: $A$
- **Singular Value of $X$**: $^*A$

#### Singular vs. Granular Values

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<th>Granular</th>
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<td>Unemployment</td>
<td>7.3%</td>
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</tr>
<tr>
<td>Probability</td>
<td>.8</td>
<td>high</td>
</tr>
<tr>
<td>Blood Pressure</td>
<td>160/80</td>
<td>high</td>
</tr>
</tbody>
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EXAMPLE

- Age as a singular variable takes values in the interval [0,120].
- *Age as a granular (linguistic) variable takes as values fuzzy subsets of [0,120] labeled young, middle-aged, old, not very young, etc.
GRANULATION OF A FUNCTION
GRANULATION = SUMMARIZATION

- If $X$ is small then $Y$ is small
- If $X$ is medium then $Y$ is large
- If $X$ is large then $Y$ is small

$f : \text{perception} \rightarrow *f$ (fuzzy graph)

$X \rightarrow Y$

Granule

Medium × Large

If $X$ is small then $Y$ is small
If $X$ is medium then $Y$ is large
If $X$ is large then $Y$ is small
KEY POINTS

- What this means is that $p^*$ should qualify to serve as an object of computation.

- In CW, precisiation is aimed at conversion of $p$ into a generalized assignment statement or, equivalently, a generalized constraint. A generalized constraint may be viewed as a model of $p$. 

$p \xrightarrow{\text{precisiation}} X \text{ isr } R$
BASIC CONCEPTS

- precisiland = model of meaning
- precisiation ≈ modelization
- intension = attribute-based meaning
- cointension = measure of proximity of meanings
  = measure of proximity of the model and the object of modelization
In CW, a proposition, p, is viewed as an answer to a question, q, of the form, q: What is the value of a variable, X, which is explicit or implicit in p? What this implies is that given p there exists a variable, X, such that p may be viewed as an answer to the question: What is the value of X?
Example:

Vera is middle-aged is an answer to the question: What is Vera’s age?

- As an answer to a question, q, a proposition, p, may be viewed as a carrier of information about X.

- More concretely, p, may be viewed as a generalized assignment statement (Zadeh 1986) which assigns a value, R, to X. R may be explicit or implicit in p.
Equivalently, a proposition, $p$, may be viewed as a generalized constraint, with $X$ and $R$ being the constrained variable and the constraining relation, respectively. The meaning of $p$ may be represented as a generalized constraint.

$$X \text{ isr } R$$

where $r$ defines the modality of the constraint, that is, the way in which $R$ constrains $X$. 
PROPOSITION AS A GENERALIZED CONSTRAINT

$p \xrightarrow{\text{isr}} X \quad R$

- $p$: constrained variable
- $X$: copula
- $R$: constraining relation

$R$ is the value of $X$

$R$ is a predicate

A generalized constraint may be viewed as a model of $p$. 
EXAMPLES

- Robert is tall $\rightarrow$ \textit{Height(Robert) is tall}
  \[ \uparrow \quad X \quad \uparrow \quad R \]
  blank (possibilistic constraint)

- Most Swedes are tall $\rightarrow$
  \textit{Count(tall.Swedes/Swedes) is most}
  \[ \uparrow \quad X \quad \uparrow \quad R \]
  blank

- Most Swedes are tall
  \[ (1\text{-most}) \text{ Swedes are not tall} \]
In CW, the meaning of a proposition is defined by $X$, $r$ and $R$.

Precisiation of $p$ involves three phases: rewording, graduation and composition.
CONTINUED

Phase 1: From q, I to q*, I*

Aggregation function: $\text{Ans}(q/I) = f(X_1, \ldots, X_n)$
Phase 2: Computation

\[ p_1^*, \ldots, p_n^* \rightarrow \text{generalized constraint propagation} \rightarrow \text{Ans}(q/l) \]
GENERALIZED CONSTRAINT (Zadeh 1986)

• Bivalent constraint (hard, inelastic, categorical:)
  \[ X \in C \]
  constraining bivalent relation

• Generalized constraint on \( X \): GC(X)  (elastic)
  GC(X): \( X \) is \( R \)

  constraining non-bivalent (fuzzy) relation
  index of modality (defines semantics)
  constrained variable

  \( r: \varepsilon | = | \leq | \geq | \subset | \ldots | \text{blank} | p | v | u | rs | fg | ps | \ldots \)

  bivalent

  primary

• open GC(X): \( X \) is free (GC(X) is a predicate)

• closed GC(X): \( X \) is instantiated (GC(X) is a proposition)
PRIVMARY GENERIALIZED CONSTRAINTS

- Possibilistic: $X$ is $R$
- Probabilistic: $X$ isp $R$
- Veristic: $X$ isv $R$

Primary constraints are formalizations of three basic perceptions: (a) perception of possibility; (b) perception of likelihood; and (c) perception of truth.

In this perspective, probability may be viewed as an attribute of perception of likelihood.
In large measure, computation with generalized constraints involves the use of rules which govern propagation and counterpropagation of generalized constraints. Among such rules, the principal rule is the Extension Principle (Zadeh 1965, 1975 a, b and c).
GENERALIZED EXTENSION PRINCIPLE

\[
f(X) \text{ is } A \\
g(X) \text{ is } B
\]

\[
\mu_B(w) = \sup_u \mu_A(f(u))
\]

subject to

\[
w = g(u)
\]

\(\mu_A\) and \(\mu_B\) are the membership functions of A and B, respectively.
STRUCTURE OF THE EXTENSION PRINCIPLE

\[ f^{-1}(A) \]

\[ \mu_A(f(u)) \]

\[ f \]

\[ g \]

\[ g(f^{-1}(A)) \]

\[ \text{counterpropagation} \]

\[ \text{propagation} \]
EXAMPLES OF APPLICATION OF THE EXTENSION PRINCIPLE TO EVERYDAY REASONING

- The Robert Problem
- Additional examples in Appendix
- **Reasoning is** $p$-valid.
- **Answer is an approximate solution to** $p$. 
TOWARD AN UNPRECISIABLE LOGIC OF EVERYDAY REASONING—A FIRST STEP
PREAMBLE

- In precisiable logic of everyday reasoning precisiation is measurement-based. In unprecisiable logic of everyday reasoning precisiation is perception-based.
- Much of everyday reasoning is perception-based.
- Unprecisiable logic of everyday reasoning is a radical departure from traditional logical systems.
In unprecisiable logic of everyday reasoning, reasoning is f-valid. There are no formal definitions, no formal rules of inference and no formal proofs.

A model for unprecisiable everyday reasoning is f-geometry.
f-VALID REASONING AND f-GEOMETRY

- In f-geometry the drawing instrument is a spray pen with an adjustable spray pattern. Drawing is done by hand.
- f-geometry is unrelated to Poston’s fuzzy geometry (Poston, 1971), coarse geometry (Roe, 1996), fuzzy geometry of Rosenfeld (Rosenfeld, 1998), fuzzy geometry of Buckley and Eslami (Buckley and Eslami, 1997), fuzzy geometry of Mayburov (Mayburov, 2008), and fuzzy geometry of Tzafestas (Tzafestas et al, 2006).
Note that fuzzy figures, as shown, are not hand drawn. They should be visualized as hand drawn figures.
Informally, in the context of f-geometry, an f-transform of C is the result of execution of the instruction: Draw C by hand with a spray pen.
**f-CONCEPTS IN f-GEOMETRY**

- **f-point**
- **f-line**
- **f-triangle**
- **f-parallel**
- **f-similar**
- **f-circle**
- **f-median**
- **f-perpendicular**

- **f-bisector**
- **f-altitude**
- **f-concurrence**
- **f-tangent**
- **f-definition**
- **f-theorem**
- **f-proof**
- **...**
The cointension of $f\text{-}C$ is a qualitative measure of the proximity of $f\text{-}C$ to its prototype, $C$. A fuzzy transform, $f\text{-}C$, is cointensive if its cointension is high. Unless stated to the contrary, $f$-transforms are assumed to be cointensive.

A key idea in $f$-geometry is the following: if $C$ is $p$-valid then its $f$-transform, $f\text{-}C$, is $f$-valid with a high validity index. As a simple example, consider the definition, $D$, of parallelism in Euclidean geometry.
**f-TRANSFORMATION OF DEFINITIONS**

D: Two lines are parallel if for any transversal that cuts the lines the corresponding angles are congruent.

- f-transform of this definition reads:
  
  \[ \text{f-D: Two f-lines are f-parallel if for any f-transversal that cuts the lines the corresponding f-angles are f-congruent.} \]
In Euclidean geometry, two triangles are similar if the corresponding angles are congruent. Correspondingly, in f-geometry two f-triangles are f-similar if the corresponding angles are f-congruent.
Simple example

- \( P: \) if the triangles \( A, B, C \) and \( A', B', C' \) are similar, then the corresponding sides are in proportion.

\[
\frac{AB}{A'B'} = \frac{BC}{B'C'} = \frac{CA}{C'A'}
\]
*P: if the f-triangles *A, *B, *C and A’, B’, C’ are f-similar, then the corresponding sides are in f-proportion.

\[ \frac{*A*B}{A'B'} = \frac{*B*C}{B'C'} = \frac{*C*A}{C'A'} \]
An f-theorem in f-geometry is an f-transform of a theorem in Euclidean geometry.

Simple example

an elementary theorem, $T$, in Euclidean geometry is:

$T$: the medians of a triangle are concurrent.

A corresponding theorem, $f-T$, in f-geometry is:

$f-T$: the f-medians of an f-triangle are f-concurrent.
A logical f-proof is an f-transform of a proof in Euclidean Geometry.
D, E are f-midpoints
DE is f-parallel to BC
FH is f-parallel to BC
AGI is an f-line passing through f-point G
f-triangles EGH and EBC are f-similar

f-triangles DFG and DBC are f-similar
f-proportionality of corresponding sides of f-triangles implies that G is f-midpoint of FH
G is f-midpoint of FH implies that I is f-midpoint of BC
I is f-midpoint of BC implies that the f-medians are f-concurrent
The f-theorem and its f-proof are f-transforms of their counterparts in Euclidean geometry. But what is important to note is that the f-theorem and its f-proof could be arrived at without any reference to their counterparts in Euclidean geometry.
A Key Observation

- This suggests an intriguing possibility of constructing, in various fields, independently arrived at systems of f-concepts, f-definitions, f-theorems, f-proofs and, more generally, f-reasoning and f-computation. In the conceptual world of such systems, p-validity has no place.
In summary, f-geometry may be viewed as the result of application of f-transformation to Euclidean geometry.

Beyond f-geometry lies an expanse of various fields to which f-transformation may be applied. Following are a few examples.
Convex Sets

D: A is a convex set in U if for any points x and y in A every point in the segment xy is in A. The f-transform of this definition is the definition of an f-convex set, f-A. Specifically,

f-D: f-A is an f-convex set in U if for any f-points x and y in f-A every f-point in the f-segment xy is in f-A.
An elementary property of convex sets is:

\[ T: \text{if } A \text{ and } B \text{ are convex sets, so is their intersection } A \cap B. \]

An f-transform of T reads:

\[ f-T: \text{if } A \text{ and } B \text{ are f-convex sets, so is their intersection } f-A \cap f-B. \]
More generally,

\[ T: \text{if A and B are convex fuzzy sets, so is their intersection.} \]

- Applying f-transformation to \( T \), we obtain the f-theorem:

\[ f-T: \text{if A and B are f-convex fuzzy sets, so is their intersection.} \]
A basic problem which arises in computation of f-transforms is the following. Let $g$ be a function, a functional or an operator. Using the star notation, let an f-transform, $^*C$, be an argument of $g$. The problem is that of computing $g(^*C)$. Generally, computing $g(^*C)$ is not a trivial problem.
An $f$-valid approximation to $g(*C)$ may be derived through application of an $f$-principle which is referred to as precisiation/imprecisiation principle or P/I principle, for short (Zadeh 2005). More specifically, the principle may be expressed as

$$g(*C) = g(C)$$

where $*$ should be read as approximately equal. In words, $g(*C)$ is approximately equal to the $f$-transform of $g(C)$. 
EXAMPLE

- If $g$ is the operation of differentiation and $^*C$ is an $f$-function, $^*f$, then the $f$-derivative of this function is an $f$-function.
In many real-world settings, an f-valid solution based on a realistic model may be better than a p-valid solution based on an unrealistic model.
EPILOGUE

- What does the logic of everyday reasoning have to offer?

- Basically, the logic of everyday reasoning is a key to automation (mechanization) of many remarkable human capabilities which involve perceptions rather than measurements.
An example is the problem of stabilization of inverted pendulum (Yamakawa 1987).
YAMAKAWA’S INVERTED PENDULUM (1989)
MATHEMATICAL MODEL (v-PRECISE)

\[ I\ddot{\theta} = V L \sin \theta - H L \cos \theta, \]
\[ V - mg = -mL(\dot{\theta} \sin \theta + \dot{\theta}^2 \cos \theta), \]
\[ H = m\ddot{y} + mL(\ddot{\theta} \cos \theta - \dot{\theta}^2 \sin \theta), \]
\[ U - H = M\ddot{y}, \]
Assignment of Membership Functions

PL : Positively Large
PM : Positively Medium
PS : Positively Small
NL : Negatively Large
NM : Negatively Medium
NS : Negatively Small
ZR : Approximately Zero

$\mu(x)$

Minimum

Maximum
**Control Rule of Velocity**

\[
X = A \cdot \theta + B \cdot (d\theta/dt) = \text{NS}
\]

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**Target Point**

- **PL:** Positively Large
- **PM:** Positively Medium
- **PS:** Positively Small
- **ZR:** Approximately Zero
- **NS:** Negatively Small
- **NM:** Negatively Medium
- **NL:** Negatively Large

**Diagram Notes:**
- U: Control Input
- \(\theta\): Angular Position
- \(x\): Linear Position
- \( \frac{dx}{dt} \): Velocity
- \( \frac{d\theta}{dt} \): Angular Velocity

**Equations:**

\[
\frac{dx}{dt} = y = C \cdot x + D \cdot (dx/dt) = \text{NS}
\]

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LINGUISTIC MODEL (PERCEPTION-BASED)

\[
\begin{align*}
\text{IF } \dot{\theta} \text{ is PM and } \dot{\theta} \text{ is ZR, then } \dot{y} \text{ is PM,} \\
\text{ALSO} \\
\text{IF } \dot{\theta} \text{ is PS and } \dot{\theta} \text{ is PS, then } \dot{y} \text{ is PS,} \\
\text{ALSO} \\
\text{IF } \dot{\theta} \text{ is PS and } \dot{\theta} \text{ is NS, then } \dot{y} \text{ is ZR,} \\
\text{ALSO} \\
\text{IF } \dot{\theta} \text{ is NM and } \dot{\theta} \text{ is ZR, then } \dot{y} \text{ is NM,} \\
\text{ALSO} \\
\text{IF } \dot{\theta} \text{ is NS and } \dot{\theta} \text{ is NS, then } \dot{y} \text{ is NS,} \\
\text{ALSO} \\
\text{IF } \dot{\theta} \text{ is NS and } \dot{\theta} \text{ is PS, then } \dot{y} \text{ is ZR,} \\
\text{ALSO} \\
\text{IF } \dot{\theta} \text{ is ZR and } \dot{\theta} \text{ is ZR, then } \dot{y} \text{ is ZR.}
\end{align*}
\]
Variable universe adaptive fuzzy control on the quadruple inverted pendulum (2002)
H. Li, Z. Miao & J-Y. Wang
APPENDIX
EXAMPLES WITH SOLUTIONS

Problem 1

- **Question:** What is the probability that Magnus is blond?
- **Information set.**
  - $p_1$: Most Swedes are tall
  - $p_2$: Most tall Swedes are blond
  - $p_3$: Magnus is a Swede (picked at random)
- **It is convenient to solve this problem through application of the intersection/product syllogism (Zadeh 1983).**
Intersection/product syllogism. \( Q_1 \) and \( Q_2 \) are fuzzy quantifiers (fuzzy numbers); \( A \), \( B \) and \( C \) are fuzzy sets.

- \( Q_1 \)A’s are B’s
- \( Q_2 \) (A and B)’s are C’s
- \( Q_1 \times Q_2 \) A’s are (B and C)’s
- at least \( Q_1 \times Q_2 \) A’s are C’s

If \( Q_1 \) and \( Q_2 \) are monotonic the last conclusion can be replaced with
- \( Q_1 \times Q_2 \) A’s are C’s.
Since most is a monotonic quantifier, and Magnus is picked at random, application of the intersection/product syllogism to the Magnus problem may be expressed as:

\[ \text{Prob(Magnus is blond) is most} \times \text{most} \]
EXAMPLE

Problem 2

- Question: What is the average height of Swedes?

- Information set. \( p \): Most Swedes are tall

- Generalized constraint representation of \( p \)

\[
\text{Count(tall.Swedes/Swedes) is most}
\]

- Discrete version

\[
P = \text{population of Swedes}
\]

\[
P = (\text{Name}_1, \ldots, \text{Name}_n)
\]
CONTINUED

$h_i$: height of Name$_i$, $i=1, \ldots, n$.

$\mu_{tall}(h_i)$: grade of membership of $h_i$ in the fuzzy set tall

Count (tall.Swedes/Swedes)=$\frac{1}{n}\Sigma_i\mu_{tall}(h_i)$

(follows from definition of Count)

Consequently,

Most Swedes are tall $\rightarrow$ $\frac{1}{n}\Sigma_i\mu_{tall}(h_i)$ is most
\( \mu_{\text{most}}: \) membership function of most

\[ \text{Average.height} = \frac{1}{n} \sum_i h_i \]

Apply the Extension Principle to

\[ \frac{1}{n} \sum_i \mu_{\text{tall}}(h_i) \text{ is most} \quad \text{(given)} \]

\[ \frac{1}{n} \sum_i h_i \text{ is Average.height} \quad \text{(needed)} \]
Continued

**solution**

\[
\mu_{\text{Average.height}}(u) = \sup_{h_1, \ldots, h_n} \mu_{\text{most}} \left( \frac{1}{n} \sum_i \mu_{\text{tall}} h_i \right)
\]

subject to

\[
u = \frac{1}{n} \sum_i h_i
\]
Continuous version

$X(h)$: height density function (not known)

$X(h)dh$: fraction of Swedes whose height is in $[h, h+dh]$, $a \leq h \leq b$

$$\int_a^b X(h)dh = 1$$
continuing...

- fraction of tall Swedes: \[ \int_a^b X(h) \mu_{tall}(h) \, dh \]
- average height of Swedes \[ \int_a^b X(h) h \, dh \]
- constraint on \( X(h) \)

\[ \int_a^b X(h) \mu_{tall}(h) \, dh \text{ is most} \]

- Apply the Extension Principle to
\[
\int_a^b X(h) \mu_{\text{tall}}(h) \, dh \quad \text{is most (given)}
\]

\[
\int_a^b X(h) \, dh \quad \text{is ? Average.height (needed)}
\]

solution:

\[
\mu_Q(v) = \sup_{\mu} \{ \mu_{\text{most}}(\int_a^b X(h) \mu_{\text{tall}}(h) \, dh) \}
\]

subject to

\[
v = \int_a^b X(h) \, dh
\]

\[
\int_a^b X(h) \, dh = 1
\]
Example

Problem 3

- **Question:** What is the difference in the average height of Swedes and the average height of Italians?
- **Information set:** Most Swedes are much taller than most Italians

**Solution**

\[ S = \{S_1, \ldots, S_n\}: \text{population of Swedes} \]
\[ I = \{I_1, \ldots, I_n\}: \text{population of Italians} \]
\[ g_i = \text{height of } S_i, \quad g = (g_1, \ldots, g_n) \]
\[ h_j = \text{height of } I_j, \quad h = (h_1, \ldots, h_n) \]
\[ \mu_{ij} = \mu_{\text{much.taller}}(g_i, h_j) = \text{degree to which } S_i \text{ is much taller than } I_j \]
$r_i = \frac{1}{n} \sum_j \mu_{ij}$ = Count of Italians in relation to whom $S_i$ is much taller

$t_i = \mu_{\text{most}}(r_i) = \text{degree to which } S_i \text{ is much taller than most Italians}$

$v = \frac{1}{m} \sum t_i$ = Count of Swedes who are much taller than most Italians

$ts(g, h) = \mu_{\text{most}}(v)$

$p \rightarrow \text{generalized constraint on } S \text{ and } I$

$q: d = \frac{1}{m} \sum_i g_i - \frac{1}{n} \sum_j h_j$
CONTINUED

Solution via the Extension Principle

\[ \mu_q(d) = \sup_{g,h} ts(g,h) \]

subject to

\[ d = \frac{1}{m} \sum_i g_i - \frac{1}{n} \sum_j h_j \]
EXAMPLE

Problem 4

● Question, q. On the average, how many cars are stolen per month in Berkeley?

● Information set, I.

  p: Usually several cars are stolen each day in Berkeley.

● Solution.

Assume that the number of cars, i, stolen each day varies from 0 to 10.
Assume that the probability that i cars are stolen, is \( p_i \), \( i=0, ..., 10 \). The \( p_i \) are not known.
Assume that the membership function of \( \mu \text{several} \) is given.
Let $X$ be a random variable which represents the number of cars stolen each day, with the understanding that $i$ is a value of $X$.

A first step toward precisiation of $p$ is that of representing $p$ as a possibilistic generalized constraint.

$p \rightarrow \text{Prob}(X \text{ is several}) \text{ is usually}$

where $(X \text{ is several})$ is a fuzzy event and usually is its fuzzy probability (fuzzy number). The membership function of usually, $\mu_{\text{usually}}$, is assumed to be given.
The probability of the fuzzy event $X$ is several may be expressed as

$$\text{Prob}(X \text{ is several}) = \sum_{i=0}^{10} p_i \mu_{\text{several}}(i)$$

At this point, the general constraint on the $p_i$ which is induced by $p$ may be written as

$$\sum_{i=0}^{10} p_i \mu_{\text{several}}(i) \text{ is usually.}$$

or equivalently, as

$$\mu_{\text{usually}}\left(\sum_{i=0}^{10} p_i \mu_{\text{several}}(i)\right)$$
This expression defines a possibility distribution of the probability distribution of the $p_i$.

- Having a possibility distribution of the probability distribution of the $p_i$ makes it possible to compute the average number of cars stolen each day. More specifically,

$$X_{\text{ave/day}} = \sum_{i=0}^{10} ip_i$$
Applying the Extension Principle, we obtain

\[
\mu_{x_{\text{ave/day}}} (v) = \text{sup } (\mu_{\text{usually}} (\sum_{i=0}^{10} p_i \mu_{\text{several}} (i)))
\]

subject to

\[
(v) = \sum_{i=0}^{10} ip_i
\]
CONTINUED

- The thirty day average may be obtained by multiplying the daily average by thirty.

- It should be noted that the solution described above expresses $\text{Ans}(q/I)$ as a fuzzy number. This fuzzy number depends on the graduations of several and usually.
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