INFORMAL EXPOSITION OF GCS CLARIFICATION DIALOGUE

- The basic ideas which underlie precisiation of meaning and, more particularly, generalized-constraint-based semantics, are actually quite simple. To bring this out, it is expedient to precede a formal exposition of GCS with an informal narrative in the form of a dialogue between Robert and Lotfi.
Robert: Lotfi, generalized-constraint-based semantics looks complicated to me. Can you explain in simple terms the basic ideas which underlie GCS?
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Lotfi: I will be pleased to do so. Let us start with an example, \( p \): Most Swedes are tall. \( p \) is a proposition. As a proposition, \( p \) is a carrier of information. Without loss of generality, we can assume that \( p \) is a carrier of information about a variable, \( X \), which is implicit in \( p \). If I asked you what is this variable, what would you say?
Robert: As I see it, \( p \) tells me something about the proportion of tall Swedes among Swedes.

Lotfi: Right. What does \( p \) tell you about the value of the variable?

Robert: To me, the value is not sharply defined. I would say it is fuzzy.

Lotfi: So what is it?

Robert: It is the word “most.”
Lotfi: You are right. So what we see is that $p$ may be interpreted as the assignment of a value “most” to the variable, $X$: Proportion of tall Swedes among Swedes.
As you can see, a basic difference between a proposition drawn from a natural language and a proposition drawn from a mathematical language is that in the latter the variable and the value assigned to it are explicit, whereas in the former the variable and the assigned value are implicit.
There is an additional difference. When \( p \) is drawn from a natural language the assigned value is not sharply defined—typically it is fuzzy, as “most” is. When \( p \) is drawn from a mathematical language the assigned value is sharply defined.

Robert: I get the idea. So what comes next?
Lotfi: There is another important point. When *p* is drawn from a natural language the value assigned to *X* is not really a value of *X*—it is a constraint (restriction) on the values which *X* is allowed to take. This suggests an unconventional definition of a proposition *p*, drawn from a natural language. Specifically, a proposition is an implicit constraint on an implicit variable.
Robert: I notice that you are talking about generalized constraints. What is a generalized constraint? Why do we need generalized constraints?

Lotfi: A generalized constraint is expressed as:

\[ X \text{ isr } R \]
where $X$ is the constrained variable, $R$ is the constraining relation—typically a fuzzy set—and $r$ is an indexical variable which defines how $R$ constrains $X$. Let me explain why the concept of a generalized constraint is needed in precisiation of meaning of a proposition drawn from a natural language.
Standard constraints are hard in the sense that they have no elasticity. In a natural language, meaning can be stretched. What this implies is that to represent meaning a constraint must have elasticity. To deal with richness of meaning, elasticity is necessary but not sufficient. Consider the proposition: Usually most flights leave on time.
What is the constrained variable and what is the constraining relation? Actually, for most propositions drawn from a natural language a large repertoire of constraints is not necessary. What is sufficient are three so-called primary constraints and their combinations. The primary constraints are: possibilistic, probabilistic and veristic.
The role of primary constraints is analogous to the role of primary colors: red, green and blue. In most cases, constraints are possibilistic. Possibilistic constraints are much easier to manipulate than probabilistic constraints.
Robert: OK, suppose I figured out what is the constrained variable, X, and the constraining relation, R. Is there something else that has to be done?

Lotfi: Yes, there is. You see, X and R are described in a natural language. What this means is that we are not through with precisiation of meaning of p. What remains to be done is precisiation (definition) of X and R.
For this purpose, we construct a so-called explanatory database, ED, which consists of a collection of relations in terms of which X and R can be defined. The entries in relations in ED are referred to as database variables. Database variables are assumed to be uninstantiated.
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Robert: Can you be more specific?

Lotfi: To construct ED you ask yourself the question: What information—in the form of a collection or relations—is needed to precisiate (define) X and R? Looking at p, we see that to precisiate X we need two relations: POPULATION.SWEDES[Name; Height] and TALL[Height; $\mu$].
In the relation \( TALL[Height; \mu] \), \( \mu_{\text{tall}}(h) \) is the grade of membership of a value of Height, \( h \), in the fuzzy set tall. So far as \( R \) is concerned, the needed relation is \( MOST[Proportion; \mu] \), where \( \mu \) is the grade of membership of a value of Proportion in the fuzzy set most.
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Equivalently, it is frequently helpful to ask the question: What is the information which is needed to assess the degree to which p is true?
At this point, we can express ED as the collection:

\[
ED = \text{POPULATION.SWEDES[Name; Height]} + \text{TALL[Height; } \mu \text{]} + \text{MOST[Proportion; } \mu \text{]}
\]

in which for convenience plus is used in place of comma.
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Robert: So, we have constructed ED for the proposition, $p$: Most Swedes are tall. More generally, given a proposition, $p$, how difficult is it to construct ED for $p$?

Lotfi: For humans it is easy. A few examples suffice to learn how to construct ED. Construction of ED is easy for humans because humans have world knowledge. At this juncture, we do not have an algorithm for constructing ED.
Robert: Now that we have ED, what comes next?

Lotfi: We can use ED to precisiate (define) X and R. Let us start with X. In words, X is described as the proportion of tall Swedes among Swedes. Let us assume that in the relation POPULATION.SWEDES there are N names. Then the proportion of tall Swedes among Swedes would be the number of tall Swedes divided by N.
Here we come to a problem. Tall Swedes is a fuzzy subset of Swedes. The question is: What is the number of elements in a fuzzy set? In fuzzy logic, there are different ways of answering this question. The simplest is referred to as the $\Sigma$Count. More concretely, if $A$ is a fuzzy set with a membership function $\mu_A$, then the $\Sigma$Count of $A$ is defined as the sum of grades of membership in $A$. 
In application to the number of tall Swedes, the $\Sigma$Count of tall Swedes may be expressed as:

$$\Sigma\text{Count(tall.Swedes)}= \sum_{i=1}^{N} \mu_{\text{tall}}(h_i)$$

where $h_i$ is the height of Name$_i$. Consequently, the proportion of tall Swedes among Swedes may be written as:

$$X = \frac{1}{N} \left( \sum_{i=1}^{N} \mu_{\text{tall}}(h_i) \right)$$
This expression may be viewed as a precisiation (definition) of $X$ in terms of ED.

Precisiation (definition) of $R$ is simpler. Specifically, $R=\text{most}$, where most is a fuzzy set. At this point, we have precisiated (defined) $X$ and $R$ in terms of ED.
Robert: So what have we accomplished?
Lotfi: We started with a proposition, p:
Most Swedes are tall. We interpreted p as a generalized (possibilistic) constraint. We identified the constrained variable, X, as the proportion of tall Swedes among Swedes. We identified the constraining relation, R, as a fuzzy set, most. Next, we constructed an explanatory database, ED.
Finally, we precisiated (defined) \( X \) and \( R \) in terms of \( ED \). In this way, we precisiated the meaning of \( p \), which was our objective. The precisiated meaning may be expressed as the constraint:

\[
\frac{1}{N} \left( \sum_{i=1}^{N} \mu_{\text{tall}}(h_i) \right) \text{ is most}
\]

Robert: So, you precisiated the meaning of \( p \). What purpose does it serve?
Lotfi: The principal purpose is the following. Unprecisiated (raw) propositions drawn from a natural language cannot be computed with. Precisiation is a prerequisite to computation. What is important is that precisiation of meaning opens the door to computation with natural language.
Robert: Sounds great. I am impressed. However, it is not completely clear to me what you have in mind when you say “opens the door to computation with natural language.” Can you clarify it?

Lotfi: With pleasure. Computation with natural language or, more or less equivalently, Computing with Words (CW or CWW), is largely unrelated to natural language processing.
More specifically, computation with natural language is focused on computation with information described in a natural language. Typically, what is involved is solution of a problem which is stated in a natural language. Let me go back to our example, p: Most Swedes are tall. Given this information, how can you compute the average height of Swedes.
Robert: Frankly, your question makes no sense to me. Are you serious? How can you expect me to compute the average height of Swedes from the information that most Swedes are tall?

Lotfi: That is conventional wisdom. A mathematician would say that the problem is ill-posed. It appears to be ill-posed for two reasons.
First, because the given information: Most Swedes are tall, is fuzzy, and second because you assume that I am expecting you to come up with a crisp answer, like the average height of Swedes is 5’10”. Actually, what I expect is a fuzzy answer—it would be unreasonable to expect a crisp answer.

Robert: Thanks for clarification. I am beginning to understand your question.
Lotfi: I should like to add a key point. The problem becomes well-posed if $p$ is precisiated. This is the essence of Computing with Words.
Robert: I am beginning to understand the need for precisiation, but my understanding is not complete as yet. Can you explain how the average height of Swedes can be computed from precisiated $p$?

Lotfi: Recall that precisiated $p$ is a possibilistic constraint expressed as:

$$\frac{1}{N} \left( \sum_{i=1}^{N} \mu_{\text{tall}}(h_i) \right)$$

is most
From the definition of a possibilistic constraint it follows that the constraint on X may be rewritten as:

\[ t = \mu_{\text{most}} \left( \frac{1}{N} \sum_{i=1}^{N} \mu_{\text{tall}}(h_i) \right) \]

What this expression means is that given the \( h_i \), \( \mu_{\text{tall}} \) and \( \mu_{\text{most}} \), we can compute the degree, \( t \), to which the constraint is satisfied.
It is this degree, \( t \), that is the truth-value of \( p \). Now, here is a key idea. The precisiated \( p \) constrains \( X \). \( X \) is a function of database variables. It follows that indirectly \( p \) constrains database variables. This has important implications. Let me elaborate.
What we see is that the constraint induced by p on the $h_i$ is of the general form

$$f(h_1, \ldots, h_N)$$

is most

What we are interested in is the induced constraint on the average height of Swedes. The average height of Swedes may be expressed as:

$$h_{ave} = \frac{1}{N} \left( \sum_{i=1}^{N} h_i \right)$$
This expression is of the general form
\[ g(h_1, \ldots, h_N) \text{ is } ?h_{\text{ave}} \]
where \(?h_{\text{ave}}\) is a fuzzy set that we want to compute.
At this stage, we can employ the Extension Principle of fuzzy logic to compute $h_{\text{ave}}$. (Zadeh 1975 I, II & III) In general terms, this principle tells us that from a given constraint of the form

$$f(x_1, \ldots, x_N) \text{ is } A$$

in which $A$ is a fuzzy set, we can derive an induced constraint on $g(x_1, \ldots, x_N)$,

$$g(x_1, \ldots, x_N) \text{ is } B,$$
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in which $B$ is a fuzzy set defined by the solution of the mathematical program

$$\mu_B(v) = \sup_{x_1, \ldots, x_N} \mu_A(f(x_1, \ldots, x_N))$$

subject to

$$v = g(x_1, \ldots, x_N)$$

In application to our example, what we see is that we have reduced computation of the average height of Swedes to the solution of the mathematical program
\[ \mu_B(v) = \sup_{h_1, \ldots, h_N} \mu_{\text{most}}(f(h_1, \ldots, h_N)) \]

subject to

\[ v = \frac{1}{N} \left( \sum_{i=1}^{N} h_i \right) \]

In effect, this is the solution to the problem which I posed to you. As you can see, reduction of the original problem to the solution of a mathematical program is not so simple.
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However, solution of the mathematical program to which the original problem is reduced is well within the capabilities of desktop computers.
Robert: I am beginning to see the basic idea. Through precisiation, you have reduced the problem of computation with information described in a natural language—a seemingly ill-posed problem—to a well-posed tractable problem in mathematical programming. I am impressed by what you have accomplished, though I must say that the reduction is nontrivial.
Without your explanation, it would be hard to see the basic ideas. I can also see why computation with natural language is a move into a new and largely unexplored territory. Thank you for clarifying the import of your statement: precisiation of meaning opens the door to computation with natural language.
Lotfi: I appreciate your comment. May I add that I believe—but have not verified it as yet—that in closed form the solution to the mathematical program may be expressed as:

\[ h_{\text{ave}} \geq \text{most} \times \text{tall} \]

where \( \text{most} \times \text{tall} \) is the product of fuzzy numbers most and tall.

Robert: This is a very interesting result, if true. It agrees with my intuition.
Lotfi: I appreciate your comment. I would like to conclude our dialogue with a prediction. As we move further into the age of machine intelligence and automated reasoning, the complex of problems related to computation with information described in a natural language, is certain to grow in visibility and importance.
The informal dialogue between Robert and Lotfi has come to an end. What follows is a formal exposition of generalized-constraint-based semantics.