# CS294-6 Lecture 11

Last Time:

1. Formulation of uncalibrated cameras

distortion on a camera is represented as a linear map

$$X' = KX, \text{ where } K = \begin{bmatrix} fS_x & S_\theta & O_x \\ 0 & fS_y & O_y \\ 0 & 0 & 1 \end{bmatrix} \text{ is the}$$

canonical form of the intrinsic parameters

and projection: $\lambda X' = K[R, T]X$.

2. Uncalibrated epipolar geometry on two images

epipolar constraint: $X_2^T \hat{T} R X_1 = 0$ only holds on calibrated images $\Rightarrow$ pin-hold camera model.

Rewrite: ① $X_2^T \hat{T} R X_1 = 0$

$$\Leftrightarrow X_2^T K^T \underbrace{K^{-T} \hat{T} R K^{-1}} K X_1 = 0$$

$$\Leftrightarrow X_2'^T F X_1' = 0$$

② $\lambda_2 X_2 = R\lambda_1 X_1 + T$

$$\Leftrightarrow \lambda_2 K X_2 = KR\lambda_1 X_1 + KT$$

$$\Leftrightarrow \lambda_2 X_2' = \lambda_1 (KRK^{-1}) X_1' + KT.$$

$$\Leftrightarrow X_2'^T \widehat{KT} (KRK^{-1}) X_1' = 0.$$

$\therefore F = K^{-T} \hat{T} R K^{-1} = \hat{T}' KRK^{-1}$. ☆

$svd(F) = U\Sigma V^T$, where $\Sigma = diag\{\sigma_1, \sigma_2, 0\}$.

3. Ambiguities:

① epipolar constraint:

$$X_2'^T F X_1' = 0 \Leftrightarrow X_2'^T \hat{T}' KRK^{-1} X_1' = 0$$

$$\Leftrightarrow X_2'^T \hat{T}' (KRK^{-1} + T'v^T) X_1' = 0$$

$\therefore$ The projection matrix $\Pi = [KRK^{-1} + T'v^T, v_4 T']$.

"four-parameter family" of ambiguities.

4. Calibration from a rig

Today.

Stratified reconstruction

1. ambiguities on perspective projection

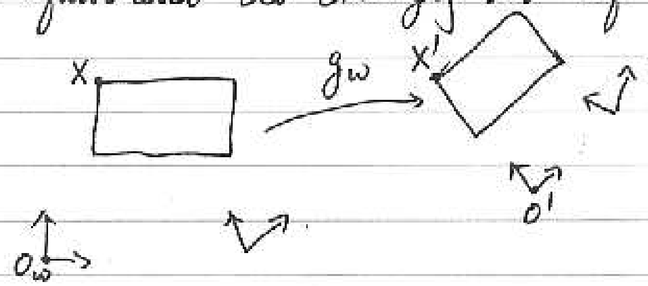
$$\lambda x' = \Pi X = K \Pi_0 g X$$
$$= \underline{K R_0^{-1}} R_0 \Pi_0 H^{-1} H g g_w^{-1} \underline{g_w X}$$

① if $X' = g_w X$.
then $gX = (g g_w^{-1}) X'$

② if $\tilde{K}' = K R_0^{-1}$
then $\lambda x' = \tilde{K} R_0 [R, T] X$
$= \tilde{K} [R_0 R, R_0 T] X. \Rightarrow \Pi' = [R_0 R, R_0 T]$

③ $\Pi'' = K \Pi_0 H^{-1}$, and.
$X_p = H g X$.

2. extrinsic parameters $g_w$.
The coordinates $X$ are expressed w.r.t. some ref. frame.
$gX = (g g_w^{-1})(g_w X)$
is equivalent to changing the ref. frame.




Then. the recovered. structure $\{X'\}$ is different from X by a Euclidean transformation.

[No sweat, choosing world coordinates is arbitrary]

3. Rotation ambiguity $R_0$
Suppose $\tilde{K}' = K R_0$, $\tilde{g} = [R_0 R, R_0 T]$. Then.
$\tilde{K}$ is a general 3x3 matrix, if $\det(K) = 1 \Rightarrow \det(\tilde{K}) = 1$

Solution: QR decomp. $qr(\tilde{K}) = K R_0$.

Notice that in terms of inner products,

$$\tilde{K}^{-T}\tilde{K}^{-1} = K^{-T}K^{-1}$$

$\Rightarrow \tilde{K}$ and $K$ generate the same distortion in the uncalibrated images.

$\therefore$ we can define an equivalence class

$$\bar{K} = K\bar{R}_0 \text{ for } R_0 \in SO(3)$$

and $\bar{K}_1 \sim \bar{K}_2$ if $\bar{K}_1 = KR_1$ for some $R_1$

$\bar{K}_2 = KR_2$ for some $R_2$.

4. Stratification:

① Three-step process:

Projective recon $\rightarrow$ Affine recon $\rightarrow$ Euclidean recon.

• Geometric viewpoint


• Algebraic viewpoint

Given two views:

$$\begin{cases} \lambda_1 x_1' = K_1 \Pi_0 g_{1e} X_e \\ \lambda_2 x_2' = K_2 \Pi_0 g_{2e} X_e \end{cases}$$

If we set the world coordinate system to be $C_1$, then $g_{1e} = [I, 0]$.

$$\lambda_1 x_1' = K_1 \Pi_0 X_e$$
$$= \underbrace{K_1 \Pi_0 H^{-1}} \underline{H X_e} \quad (\text{Ambiguity})$$
$$= \Pi_{1p} X_p.$$

Similarly, $\Pi_{2p} = K_2 \Pi_0 g_{2e} H^{-1}$, where $H \in \mathbb{R}^{4\times4}$, full-rank.

Hence, $\boxed{X_p = H X_e}$

Since $H$ is arbitrary, we can fix the form to be.

$$H^{-1} = \begin{bmatrix} K_1^{-1} & 0 \\ v^T & v_4 \end{bmatrix} \in \mathbb{R}^{4\times 4}, \text{ then}$$

$$\Pi_{1p} = K_1 [I, 0] H^{-1} = [I, 0]$$

$$\Pi_{2p} = K_2 \Pi_0 g_{2e} H^{-1}$$

$$= K_2 \Pi_0 g_{2e} \underbrace{\begin{bmatrix} K_1^{-1} & 0 \\ 0 & 1 \end{bmatrix}}_{H_a^{-1}} \underbrace{\begin{bmatrix} I & 0 \\ v^T & v_4 \end{bmatrix}}_{H_p^{-1}}$$

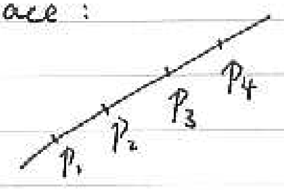
$$\Rightarrow \begin{cases} \lambda_1 x_1' = \Pi_0 X_p. \\ \lambda_2 x_2' = \Pi_{2p} X_p = K_2 \Pi_0 g_{2e} H_a^{-1} H_p^{-1} X_p. \end{cases}$$

And $X_p = H_p \underbrace{H_a X_e}_{X_a}$.

- In Summary:
  - a projective camera:
    $$\Pi_{ip} \doteq K_i \Pi_0 g_{ie} H_a^{-1} H_p^{-1}$$
  - a affine camera:
    $$\Pi_{ia} \doteq K_i \Pi_0 g_{ie} H_a^{-1}$$
  - a Euclidean camera:
    $$\Pi_{ie} \doteq K_i \Pi_0 g_{ie}$$

Sidesteps:

- Invariants in a projective space:
  ① cross ratio:
  $$Cr(p_1, p_2; p_3, p_4) = \frac{\Delta_{13} \Delta_{24}}{\Delta_{14} \Delta_{23}}$$




- Invariants in a affine space:
  ① simple ratio: $\frac{\|p_1 p_2\|}{\|p_2 p_3\|}$


$\Rightarrow$ preserves mid points

  ② parallelism