Vector Potential (a review)

$$\begin{cases} \nabla \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t} \\ \nabla \times \vec{H} = \vec{J} + \dfrac{\partial}{\partial t}\vec{D} \\ \nabla \cdot \vec{D} = \rho \\ \nabla \cdot \vec{B} = 0 \end{cases}$$ Maxwell's Eqs.

Since $\nabla \cdot (\nabla \times \vec{A}) = 0$

$\Rightarrow \vec{B} = \nabla \times \vec{A}$

$\uparrow$ vector potential

$\vec{A}' = \vec{A} + \nabla \xi$

$\nabla \times \vec{A}' = \vec{B}$ vector potential not unique

$$\nabla \times \vec{E} = -\frac{\partial}{\partial t} \nabla \times \vec{A} \Rightarrow \nabla \times (\underbrace{\vec{E} + \frac{\partial}{\partial t}\vec{A}}_{-\nabla\phi}) = 0$$

$$\vec{E} = -\frac{\partial}{\partial t}\vec{A} - \nabla\phi$$

Wave equations:

$$(\nabla^2 \vec{A} - \mu\varepsilon \frac{\partial^2}{\partial t^2}\vec{A}) - \nabla(\nabla \cdot \vec{A} + \mu\varepsilon \frac{\partial \phi}{\partial t}) = -\mu \vec{J}$$

$$\nabla^2 \phi + \frac{\partial}{\partial t}(\nabla \cdot \vec{A}) = -\frac{\rho}{\varepsilon}$$

Gauge transformation

$$\begin{cases} \vec{A}' = \vec{A} + \nabla \xi \quad \leftarrow xi \\ \phi' = \phi - \dfrac{\partial \xi}{\partial t} \end{cases}$$ also satisfy the wave eqs.

Lorentz Gauge

$$\nabla \cdot \vec{A} = -\mu\varepsilon \frac{\partial \phi}{\partial t}$$

$$\Rightarrow \begin{cases} \nabla^2 \vec{A} - \mu\varepsilon \dfrac{\partial^2}{\partial t^2}\vec{A} = -\mu \vec{J} \\ \nabla^2 \phi - \mu\varepsilon \dfrac{\partial}{\partial t^2}\phi = -\dfrac{\rho}{\varepsilon} \end{cases}$$

Coulomb Gauge

$$\nabla \cdot \vec{A} = 0$$

$$\nabla^2 \phi = -\frac{\rho}{\epsilon}$$

For optical field. $\rho = 0$, $\phi = 0$

$$\vec{E} = -\frac{\partial \vec{A}}{\partial t}$$