Semiconductor Electronics

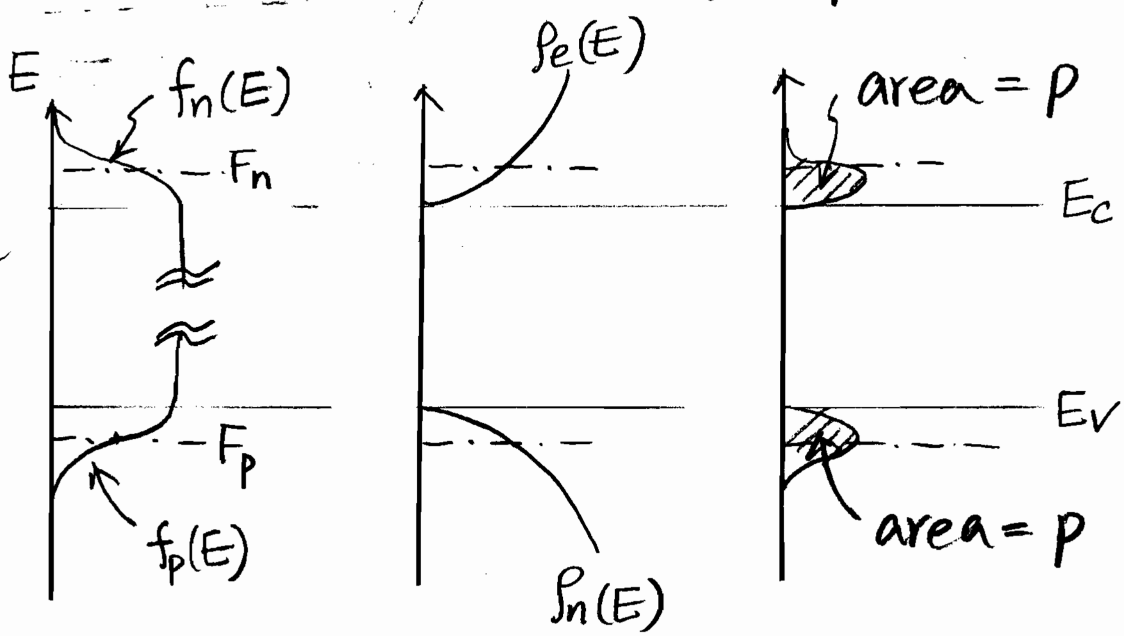
Maxwell's equations + continuity equation

For most semiconductors $H \approx 0$, $B \approx 0$

$$n = \int_{E_c}^{\infty} f_n(E) \cdot \rho_e(E)\, dE$$

$$p = \int_{-\infty}^{E_v} f_p(E) \cdot \rho_n(E)\, dE$$

f_n, f_p : Fermi-Dirac Distribution



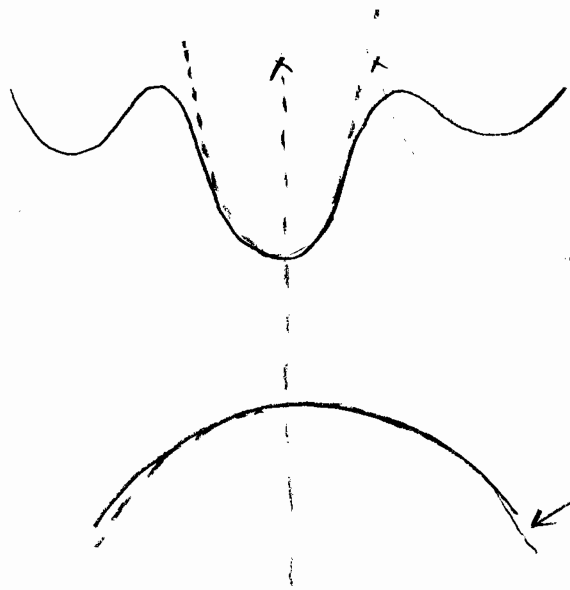
$$f_n(E) = \frac{1}{1 + e^{(E - F_n)/k_B T}}$$

$$f_p(E) = \frac{1}{1 + e^{(F_p - E)/k_B T}}$$

F_n, F_p : Quasi-Fermi levels

$\rho_e(E)$: electron density of states
$\rho_h(E)$: hole " " "

Effective mass approximation

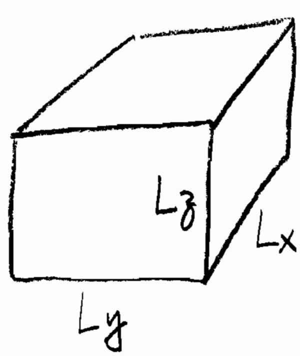


Parabolic band

$$E = E_c + \frac{\hbar^2 k^2}{2 m_e^*}$$

$$E = E_V - \frac{\hbar^2 k^2}{2 m_h^*}$$

Electron state:



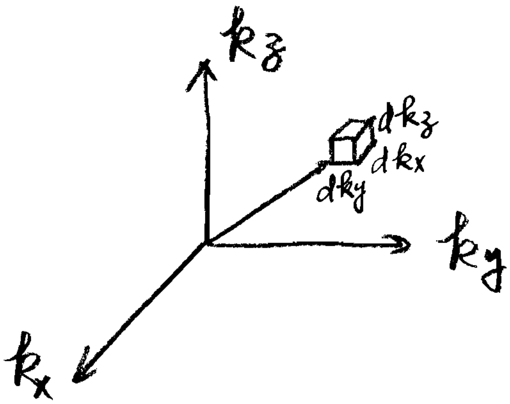
Periodic boundary condition

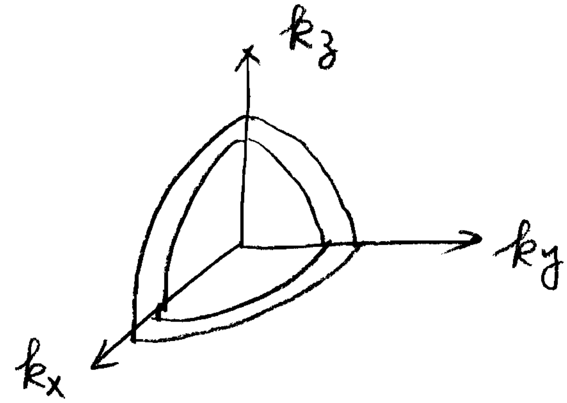
$$e^{i\vec{k}\cdot\vec{r}} = e^{i\vec{k}\cdot(\vec{r}+L_x\cdot\hat{x})} = e^{i\vec{k}\cdot(\vec{r}+L_y\cdot\hat{y})} = e^{i\vec{k}\cdot(\vec{r}+L_z\cdot\hat{z})}$$

$$k_x \cdot L_x = m \cdot 2\pi$$

$$\Rightarrow \quad k_x = m\cdot\frac{2\pi}{L_x}, \quad k_y = n\,\frac{2\pi}{L_y}, \quad k_z = l\,\frac{2\pi}{L_z}$$

Eeach electron state is represented by
(k_x, k_y, k_z) + spin (↑ or ↓)





The number of states in a cube in k-space

$$2 \times \frac{dk_x \, dk_y \, dk_z}{\left(\frac{2\pi}{L_x}\right)\left(\frac{2\pi}{L_y}\right)\left(\frac{2\pi}{L_z}\right)} = \frac{2V}{(2\pi)^3} d^3k$$

In parabolic band approximation, electrons with $|\vec{k}| = \text{const}$ have the same energy

$\rightarrow$ constant energy surface = sphere in k-space

The total number of states in a shell from $k \rightarrow k + \Delta k$ per unit volume is

$$2 \times \frac{1}{\cancel{V}} \frac{4\pi k^2 dk}{\frac{(2\pi)^3}{\cancel{V}}} = \frac{k^2}{\pi^2} dk$$

$$E = E_c + \frac{\hbar^2 k^2}{2m_e^*} \Rightarrow k = \frac{\sqrt{2m_e^*(E - E_c)}}{\hbar}$$

$$dE = \frac{\hbar^2}{m_e^*} k \, dk \Rightarrow \frac{dk}{dE} = \frac{m_e^*}{\hbar^2 k}$$

$$\int \frac{k^2 dk}{\pi^2} = \int \rho_e(E) \, dE \Rightarrow \rho_e(E) = \underbrace{\frac{k^2}{\pi^2} \frac{dk}{dE}}_{\text{expressed in } E}$$

$$\rho_e(E) = \frac{k^2}{\pi^2} \cdot \frac{m_e^*}{\hbar^2 k} = \frac{m_e^*}{\hbar^2 \pi^2} \cdot k = \frac{m_e^*}{\hbar^2 \pi^2} \cdot \frac{\sqrt{2 m_e^*}}{\hbar} \cdot \sqrt{E - E_c}$$

$$\rho_e(E) = \frac{1}{2\pi^2} \left(\frac{2 m_e^*}{\hbar^2} \right)^{3/2} \cdot \sqrt{E - E_c} \quad \text{for } E > E_c$$

Likewise

$$\rho_h(E) = \frac{1}{2\pi^2} \left(\frac{2 m_h^*}{\hbar^2} \right)^{3/2} \sqrt{E_v - E} \quad \text{for } E < E_v$$

$$n = \int_{E_c}^{\infty} \left[\frac{1}{2\pi^2} \left(\frac{2 m_e^*}{\hbar^2} \right)^{3/2} \right] \cdot \frac{(E - E_c)^{1/2}}{1 + e^{(E - F_n)/k_B T}} \cdot dE$$

$$x = \frac{E - E_c}{k_B T} \qquad dE = (k_B T)\, dx$$

$$n = \frac{1}{2\pi^2} \left(\frac{2 m_e^*}{\hbar^2} \right)^{3/2} \int_0^{\infty} \frac{(k_B T)^{1/2} \cdot x^{1/2}}{1 + e^{x - \frac{F_n - E_c}{k_B T}}} \cdot (k_B T)\, dx$$

$$= \frac{1}{2\pi^2} \left(\frac{2 m_e^* k_B T}{\hbar^2} \right)^{3/2} \int_0^{\infty} \frac{x^{1/2}}{1 + e^{x - \eta}} \cdot dx$$

$$\eta = \frac{F_n - E_c}{k_B T}$$

Fermi-Dirac Integral

$$F_j(\eta) = \frac{1}{\Gamma(j+1)} \cdot \int_0^{\infty} \frac{x^j}{1 + e^{(x - \eta)}} dx$$

Gamma function $\Gamma(\frac{3}{2}) = \frac{\sqrt{\pi}}{2}$

$$n = \frac{1}{2\pi^2}\left(\frac{2m_e^* k_B T}{\hbar^2}\right)^{3/2} \cdot \frac{\sqrt{\pi}}{2} \cdot F_{1/2}\left(\frac{F_n - E_c}{k_B T}\right)$$

$$n = N_c \cdot F_{\frac{1}{2}}\left(\frac{F_n - E_c}{k_B T}\right)$$

$$N_c = 2\left(\frac{m_e^* k_B T}{2\pi \hbar^2}\right)^{3/2} \approx 2.51 \times 10^{19}\left(\frac{m_e^*}{m_0} \cdot \frac{T}{300}\right)^{3/2} \frac{1}{cm^3}$$

Likewise

$$p = N_v \cdot F_{\frac{1}{2}}\left(\frac{E_v - F_p}{k_B T}\right)$$

$$N_v = 2\left(\frac{m_h^* k_B T}{2\pi \hbar^2}\right)^{3/2} \approx 2.51 \times 10^{19}\left(\frac{m_h^*}{m_0} \cdot \frac{T}{300}\right)^{3/2} \frac{1}{cm^3}$$

Approximation

$$F_{\frac{1}{2}}(\eta) \sim \begin{cases} e^{\eta} & \text{when } \eta \ll 1 \quad (\text{or } F_n \ll E_c) \\ \frac{4}{3}\left(\frac{\eta^3}{\pi}\right)^{1/2} & \text{when } \eta \gg 1 \quad (\text{or } F_n \gg E_c) \end{cases}$$

Degenerate

$\eta \ll 1$ or $F_n \ll E_c$

$$\frac{1}{1 + e^{(E - F_n)/kT}} \approx e^{-\frac{E - F_n}{kT}}$$ Boltzmann approximation