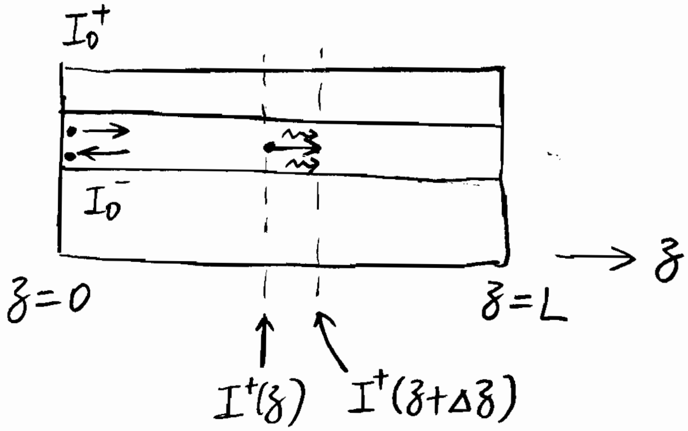
# Laser as Noise Amplifier




$$I^+(z+\Delta z) = I^+(z)\cdot e^{G_n \Delta z} + \frac{1}{2}\beta \cdot r^{spon}(\hbar\omega)\cdot \hbar\omega \cdot \Delta z$$

$\frac{1}{2}$: Forward or backward direction

$\beta$: spontaneous coupling factor: fraction of sponteneous coupled to lasing mode

For small $\Delta z$, $G_n \Delta z \ll 1$

$$I^+(z+\Delta z) = I^+(z)\,(1+G_n\Delta z) + \frac{1}{2}\beta\cdot r^{spon}(\hbar\omega)\cdot\hbar\omega\cdot\Delta z$$

$$\Rightarrow \frac{dI^+(z)}{dz} = \frac{I^+(z+\Delta z) - I^+(z)}{\Delta z} = G_n I^+(z) + \underbrace{\frac{1}{2}\beta\, r^{spon}(\hbar\omega)\cdot\hbar\omega}_{W(\hbar\omega)}$$

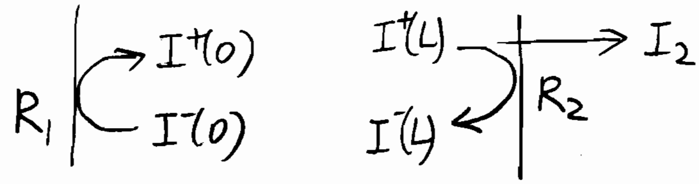
Similarly for backward wave

$$\frac{dI^-(z)}{dz} = -G_n I^-(z) - W(\hbar\omega)$$

Solutions:

$$\begin{cases} I^+(z) = I_0^+ e^{G_n\cdot z} - \dfrac{W(\hbar\omega)}{G_n(\hbar\omega)} \\ I^-(z) = I_0^- e^{-G_n z} - \dfrac{W(\hbar\omega)}{G_n(\hbar\omega)} \end{cases}$$

Boundary conditions:




$$\begin{cases} I^+(0) = R_1 I^-(0) \\ I^-(L) = R_2 I^+(L) \end{cases} \Rightarrow \text{solve } I_0^+ \text{ and } I_0^-$$

$$I_0^+ = \frac{(1+R_1 e^{G_n L}) - R_1(1+R_2 e^{G_n L})}{1-R_1R_2 e^{2G_n L}} \cdot \frac{W(\hbar\omega)}{G(\hbar\omega)}$$

Output

$$I_2 = (1-R_2)\cdot I^+(L)$$

$$= (1-R_2)\cdot \frac{(e^{G_n L}-1)(R_1 e^{G_n L}-1)}{1-R_1R_2\cdot e^{2G_n L}} \cdot \frac{W(\hbar\omega)}{G(\hbar\omega)}$$

Threshold condition

Denominator $= 0 \Rightarrow 1 - R_1R_2 e^{2G_n L} = 0$

Because of the presence of spontaneous emission $W(\hbar\omega)$, round-trip gain $R_1R_2 e^{2G_n L} < 1$

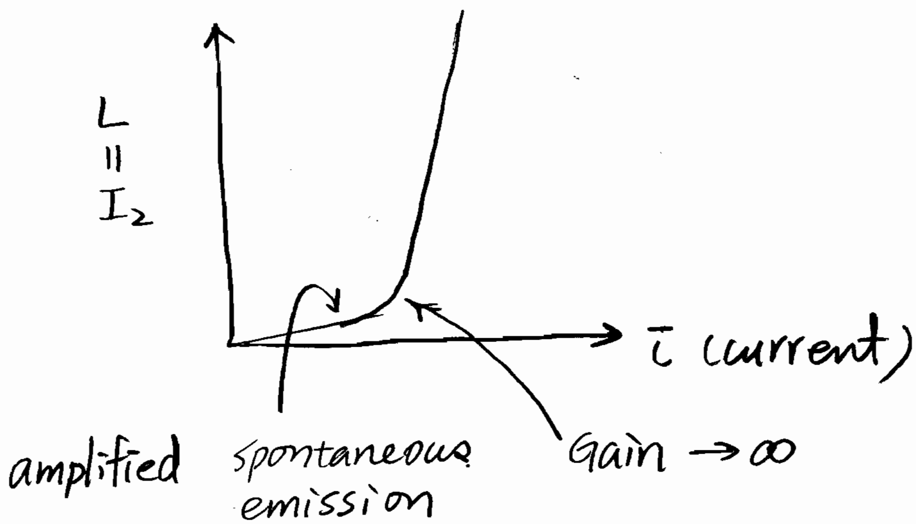
$\Rightarrow$ Denominator $> 0$

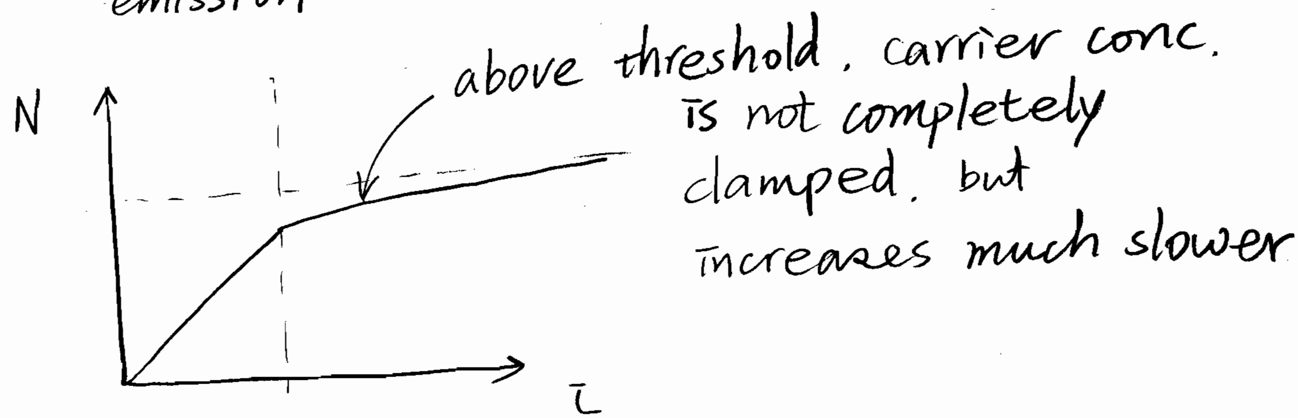
Laser is a <u>noise</u> amplifier

$\uparrow$

$W(\hbar\omega)$

# Refined model for L-I curve



# Longitudinal Modes (§7.6)

$$\vec{E}(x,y,z) = \vec{E}_0(x,y)\cdot e^{ik_z\cdot z}$$

Round-trip phase condition

$$e^{i2k_zL} = 1$$

$$2k_zL = 2\pi\cdot m \quad ; \ m: \text{integer}$$

$$k_z = \frac{\omega}{c}\cdot n_1 \qquad n_1: \text{effective refractive index}$$

$$2\cdot\frac{2\pi f_m}{c}\cdot n_1 L = 2\pi\cdot m$$

$$f_m = m\cdot\frac{c}{2n_1L} \qquad \text{Longitudinal modes.}$$

Mode spacing

$$\Delta f = f_m - f_{m-1} = \frac{c}{2n_1L} + m\cdot\frac{c}{2L}\cdot\frac{(-1)}{n_1^2}\cdot\frac{\partial n_1}{\partial f}\cdot\Delta f$$

$$\Rightarrow \Delta f = \frac{c}{2n_1L}\cdot\Big(1+\underbrace{\frac{f}{n_1}\frac{\partial n_1}{\partial f}}_{\text{material dispersion}}\Big)^{-1}$$

Or in terms of $\lambda$

$$m\cdot\lambda = 2n_1L$$

$$\Delta\left(m = \frac{2n_1L}{\lambda}\right) \Rightarrow 1 = \frac{-1}{\lambda^2}\Delta\lambda\cdot 2n_1L + \frac{2L}{\lambda}\cdot\frac{\partial n_1}{\partial\lambda}\Delta\lambda$$

$$\Delta\lambda = \frac{-\lambda^2}{2n_1L\left(1-\frac{\lambda}{n_1}\cdot\frac{\partial n_1}{\partial\lambda}\right)}$$

# Optical Spectrum

Modes

Gain

$R_1R_2\,e^{2G_n(\lambda)L}$

Amplification Factor

$$\frac{1}{1-R_1R_2\,e^{2G_n L}}$$

Lasing Spectrum

Below Threshold