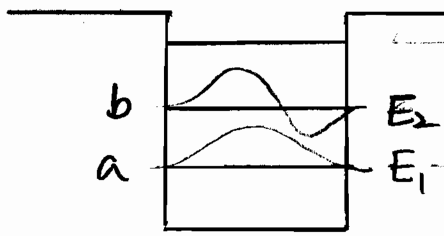
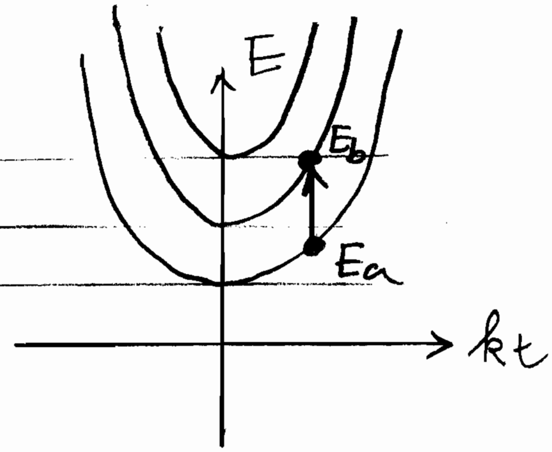
Intersubband Absorption





Infrared detector (3~5 um, 8~10 um)

Gain : Quantum Cascade laser (QCL)

$$\begin{cases} \psi_a(\vec{r}) = u_c(\vec{r}) \cdot \dfrac{e^{i\vec{k}_t\cdot\vec{\rho}}}{\sqrt{A}} \phi_1(z) \\ \psi_b(\vec{r}) = u_{c'}(\vec{r}) \dfrac{e^{i\vec{k}_t'\cdot\vec{\rho}}}{\sqrt{A}} \phi_2(z) \end{cases}$$

Note that $\langle u_c | u_{c'} \rangle \approx 1$, $\langle \phi_2 | \phi_1 \rangle = 0$

(In inter-band case $\langle u_c | u_v \rangle = 0$)

$$\vec{\mu}_{ba} = \langle \psi_b(\vec{r}) | e\vec{r} | \psi_a(\vec{r}) \rangle$$

$$\cong \langle u_c | u_{c'} \rangle \cdot \int \underbrace{\frac{e^{-i\vec{k}_t'\cdot\vec{\rho}}}{\sqrt{A}} \frac{e^{i\vec{k}_t\cdot\vec{\rho}}}{\sqrt{A}}}_{\delta_{\vec{k}_t,\vec{k}_t'}} \cdot \phi_2^*(z) \cdot e\vec{r} \cdot \phi_1(z)\, d\vec{r}$$

$$\approx \delta_{\vec{k}_t,\vec{k}_t'} \cdot \langle \phi_2 | e\vec{r} | \phi_1 \rangle$$

anti-symm in z

$\langle \phi_2 | x | \phi_1 \rangle = 0$

$\langle \phi_2 | y | \phi_1 \rangle = 0$

$$= \delta_{\vec{k}_t,\vec{k}_t'} \cdot \langle \phi_2 | ez | \phi_1 \rangle \cdot \hat{z}$$

$$= \delta_{\vec{k}_t,\vec{k}_t'} \cdot \mu_{21} \hat{z}$$

$$E_a = E_1 + \frac{\hbar^2 k_t^2}{2m_e^*}$$

$$E_b = E_2 + \frac{\hbar^2 k_t'^2}{2m_e^*}$$

$$g(\Delta E) = \frac{\left(\frac{\Gamma}{2}\right)}{(\Delta E)^2 + \left(\frac{\Gamma}{2}\right)^2}$$ Lorentzian Lineshape

$$\alpha(\hbar\omega) = \frac{\omega}{n_r c \epsilon_0} \cdot \frac{2}{V} \sum_{\vec{k}_t} \sum_{\vec{k}_t'} g(E_b - E_a - \hbar\omega) \cdot \underbrace{|\hat{e}\cdot\vec{\mu}_{ba}|^2}_{\delta_{\vec{k}_t, \vec{k}_t'} \cdot |\mu_{21}|^2} \cdot (f_a - f_b)$$

$$\hat{e}\cdot\vec{\mu}_{ba} = \underbrace{\hat{e}\cdot\hat{z}}_{\text{None zero when } \hat{e} = \hat{z} \text{ (TM)}} \cdot \mu_{21} \cdot \delta_{\vec{k}_t, \vec{k}_t'}$$

$$\alpha(\hbar\omega) = \frac{\omega}{n_r c \epsilon_0} \cdot \frac{2}{V} \sum_{\vec{k}_t} g(E_2 - E_1 - \hbar\omega) \cdot |\mu_{21}|^2 \cdot (f_a - f_b)$$

Indep of $\vec{k}_t$

$$= \frac{\omega}{n_r c \epsilon_0} \cdot g(E_{21} - \hbar\omega) \cdot |\mu_{21}|^2 \cdot \underbrace{\frac{2}{V} \sum_{\vec{k}_t} (f_a - f_b)}_{= N_1 - N_2}$$

$$= \frac{\omega}{n_r c \epsilon_0} \cdot g(E_{21} - \hbar\omega) \cdot |\mu_{21}|^2 \cdot (N_1 - N_2)$$

$$N_i = \frac{m_e^* k_B T}{\pi \hbar^2 L_z} \cdot \ln\left(1 + e^{\frac{E_F - E_i}{k_B T}}\right) \qquad i = 1, 2$$

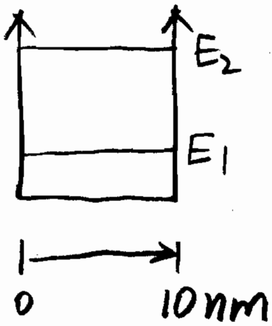
Low temp limit: $(E_F - E_i) \gg k_B T$

$$N_i \approx \frac{m_e^*}{\pi \hbar^2 L_z} \cdot (E_F - E_i) \qquad i = 1, 2$$

When $E_F \ll E_2$, $N_2 \ll N_1$, $\alpha(\hbar\omega) \propto N_1$

When $E_F \gg E_2$, $\alpha(\hbar\omega) \propto (N_1 - N_2) \propto (E_2 - E_1) = \text{const.}$

$$\alpha(\hbar\omega) = \frac{\omega}{n_c c \epsilon_0} g(E_{21} - \hbar\omega) \, |\mu_{21}|^2 \left(\frac{m_e^*}{\pi\hbar^2 L_z}\right)(E_2 - E_1)$$



GaAs

$m_e^* = 0.067 \, m_0$

$$E_1 = \frac{\hbar^2}{2m_e^*} \cdot \left(\frac{\pi}{L_z}\right)^2 = 56.5 \text{ meV}$$

$$E_2 = 4E_1 = 226 \text{ meV}$$

$$\phi_1 = \sqrt{\frac{2}{L_z}} \cdot \sin\left(\frac{\pi}{L_z} z\right)$$

$$\phi_2 = \sqrt{\frac{2}{L_z}} \sin\left(2\frac{\pi}{L_z} z\right)$$

$$\mu_{21} = e \cdot \int_0^{L_z} \phi_2(z) \cdot z \cdot \phi_1(z) \, dz$$

$$= \frac{2e}{L_z} \int_0^{L_z} \sin\left(\frac{\pi}{L_z} z\right) \cdot z \, \sin\left(2\frac{\pi}{L_z} z\right) dz$$

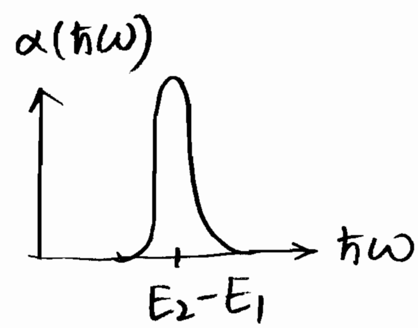
$$= -\frac{16}{9\pi^2} e L_z$$

If $N = 10^{18} \text{ cm}^{-3}$

Assume E_1 is occupied. E_2 not.

$$E_F - E_1 = N \frac{\pi\hbar^2 L_z}{m_e^*} = 36 \text{ meV}$$

Indeed E_2 is not occupied.



Peak absorption

$$\hbar\omega = E_2 - E_1 \cong 170 \text{ meV}$$

$$\lambda = \frac{1.24}{0.17} = 7.3 \, \mu\text{m}$$

$$\alpha_{peak} = \frac{\omega}{n_r c \epsilon_0} \cdot \frac{|\mu_{21}|^2}{\left(\frac{\Gamma}{2}\right)} \underbrace{(N_1 - N_2)}_{\substack{\approx \\ N}} \cong 10^4 \ cm^{-1}$$

Note: absorption $\propto N$