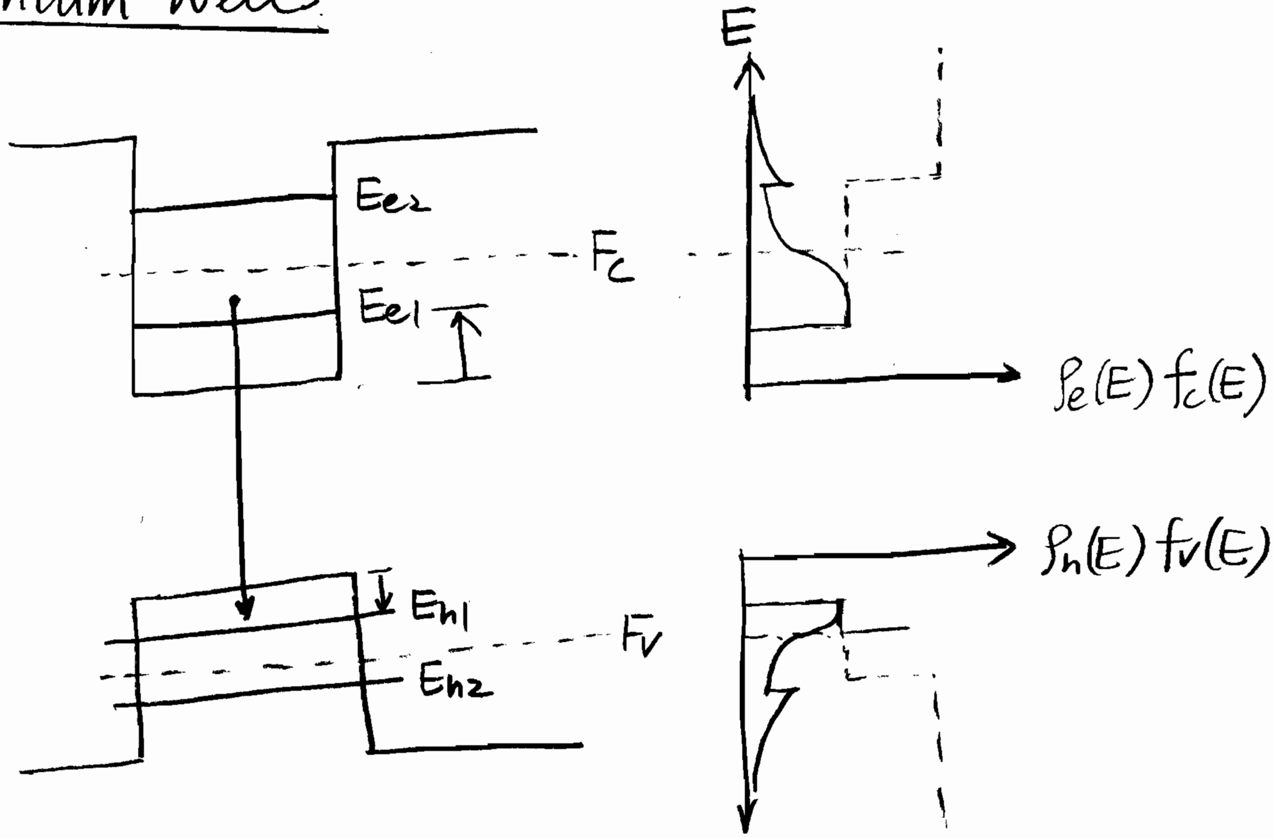
Quantum Wells



$$\alpha(\hbar\omega) = \frac{\pi\omega}{n_r c \epsilon_0} \frac{2}{V} \sum_{k_a} \sum_{k_b} |\hat{e}\cdot\vec{\mu}_{ba}|^2 \, \delta(E_b - E_a - \hbar\omega)\,(f_a - f_b)$$

* Textbook use

$$C_0 \frac{2}{V} |\hat{e}\cdot\vec{P}_{ba}|^2 = \frac{\pi\omega}{n_r c \epsilon_0} \frac{2}{V} |\hat{e}\cdot\vec{\mu}_{ba}|^2$$

$$\psi_a(\vec{r}) = \underbrace{u_v(\vec{r})}_{\text{Atomic wavefunction}} \cdot \underbrace{\frac{e^{i\vec{k}_t\cdot\vec{\rho}}}{\sqrt{A}}}_{\text{in-plane ''}} \cdot \underbrace{g_m(z)}_{\text{Envelop wavefunction in QW}}$$

A: area for normalization.

$$\psi_b(\vec{r}) = u_c(\vec{r}) \cdot \frac{e^{i\vec{k}_t'\cdot\vec{\rho}}}{\sqrt{A}} \cdot \phi_n(z)$$

$$\vec{P}_{ba} = \langle \psi_b | \vec{P} | \psi_a \rangle$$

$$\approx \langle u_c | \vec{P} | u_v \rangle \cdot \delta_{\vec{k}_t, \vec{k}_t'} \cdot I_{hm}^{en}$$

en ← n^{th} electron level

hm ← m-th hole level

$$I_{hm}^{en} = \int_{-\infty}^{\infty} dz \cdot \phi_n^*(z)\, g_m(z)$$

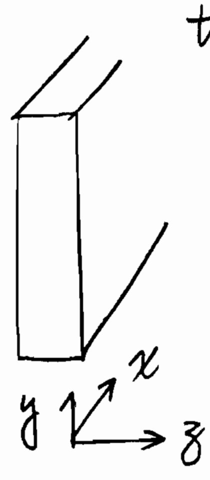
↑ overlap integral of QW envelop wavefunction

↓

Integral separated because of slowly varying envelop approx.

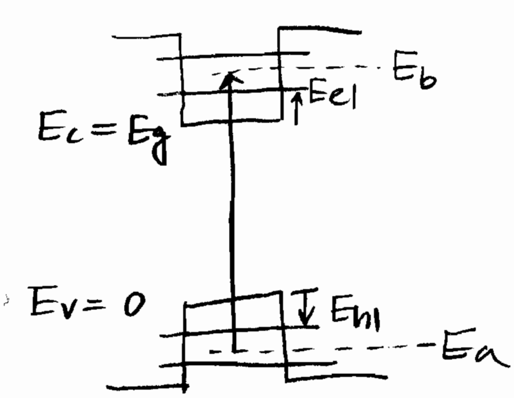
$$\frac{\text{well width}}{\text{atomic spacing}} \sim \frac{10\ \text{nm}}{.25\ \text{nm}} \sim 40$$

$\vec{k}_t = \vec{k}_{t'}$: conservation of momentum in the plane of QW



$$\vec{k}_t = k_{tx}\hat{x} + k_{ty}\hat{y}$$

$$\vec{k}_t \perp \hat{z}$$



$$E_b = E_g + E_{en} + \frac{\hbar^2 k_t^2}{2m_e^*}$$

$$E_a = 0 + E_{h1} - \frac{\hbar^2 k_t^2}{2m_h^*}$$

* Note $E_{h1} < 0$ in this convention

$$E_b - E_a = E_{hm}^{en} + \frac{\hbar^2 k_t^2}{2m_r^*}$$

separation between electron and hole quantized energy levels

For $n=1$, $m=1$

$$\alpha(\hbar\omega) = C_0 \frac{2}{V} \underbrace{\sum_{k_{at}} \sum_{k_{b,t}}}_{\sum_{k_t}} |\hat{e}\cdot\vec{P}_{ba}|^2 \cdot \delta(E_{h1}^{e1}(k_t) - \hbar\omega)(f_v - f_c)$$

$$\sum_{k_t} \quad \because \vec{k}_{at} = \vec{k}_{bt}$$

$$\frac{2}{V}\sum_{k_t} \rightarrow \int_0^\infty \rho_r^{2D}(E_t)\cdot dE_t \quad , \quad E_t = \frac{\hbar^2 k_t^2}{2m_r}$$

$$\rho_r^{2D} = \frac{m_r^*}{\pi\hbar^2 \cdot L_z}$$: Joint D.O.S. for QW

$$\alpha(\hbar\omega) = C_0 |I_{h1}^{e1}|^2 |\hat{e}\cdot\vec{P}_{cv}|^2 \rho_r^{2D} \cdot H(\hbar\omega - E_{h1}^{e1})\cdot(f_v - f_c)$$

For multiple electron (n) and hole (m) levels

$$\Rightarrow \alpha(\hbar\omega) = C_0 \cdot \sum_{n,m} |I_{hm}^{en}|^2 \cdot |\hat{e}\cdot\vec{P}_{cv}|^2 \cdot \rho_r^{2D} \cdot H(\hbar\omega - E_{hm}^{en})\cdot(f_v - f_c)$$

Population Factor in Interband Transition in QW

Eq. (9.4.1.a)

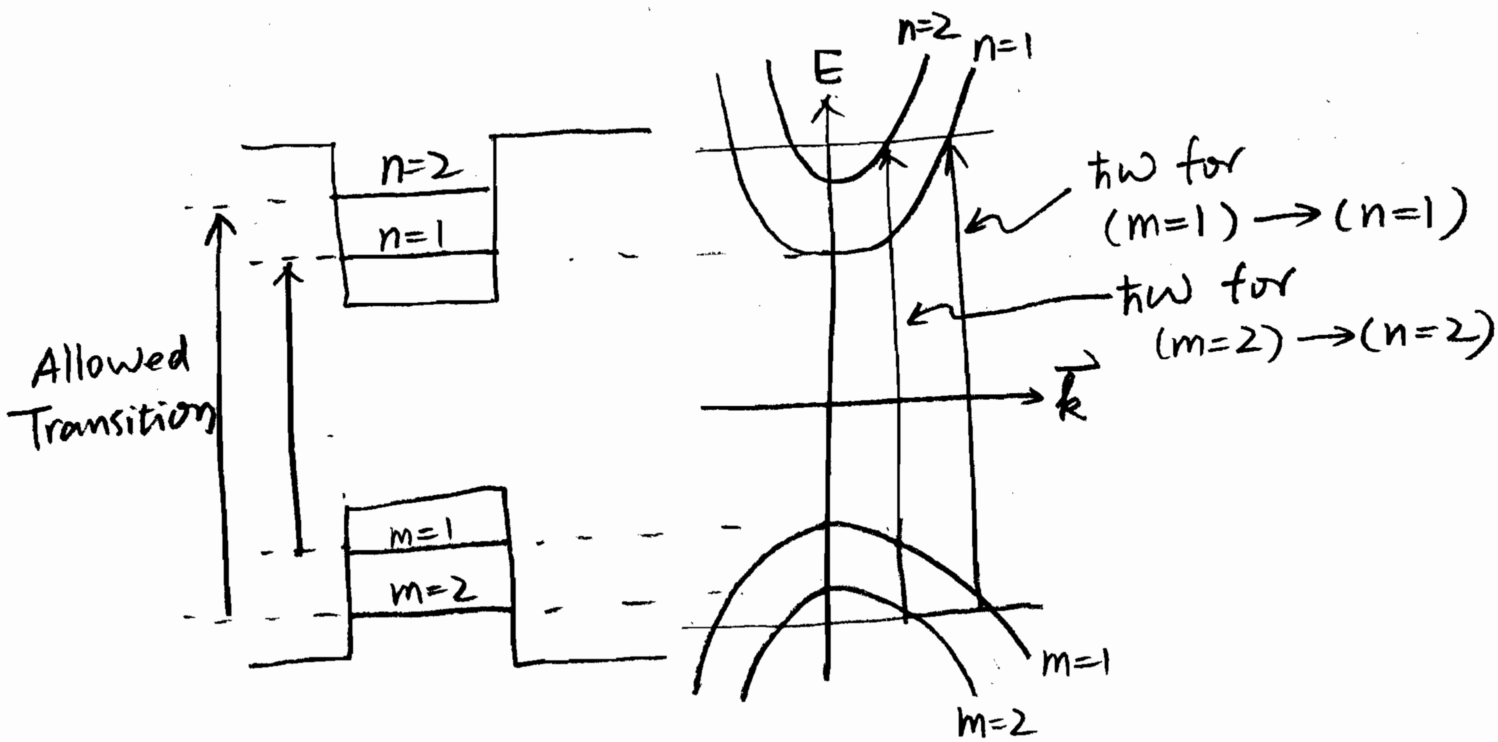
$$\alpha(\hbar\omega) = C_0 \frac{2}{V} \sum_{k_a} \sum_{k_b} |\hat{e}\cdot\vec{P}_{ba}|^2 \cdot \delta(E_{ba}-\hbar\omega)\cdot(f_v(E_a)-f_c(E_b))$$

$$= C_0 \frac{2}{V} \sum_{m,n} I_{hm}^{en} \sum_{k} |\hat{e}\cdot\vec{P}_{cv}|^2 \delta(E_{ba}-\hbar\omega)(f_v(E_a)-f_c(E_b))$$

$$= C_0 \sum_{m,n} I_{hm}^{en} \cdot \int dE_{ba} \cdot \rho_r^{2D} |\hat{e}\cdot\vec{P}_{cv}|^2 \cdot \delta(E_{ba}-\hbar\omega)(f_v(E_a)-f_c(E_b))$$

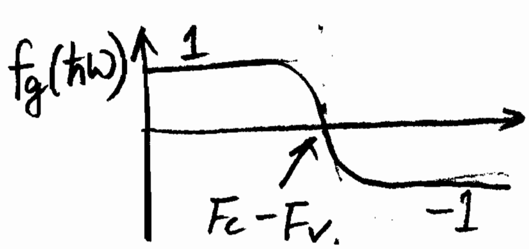
Note that $I_{hm}^{en} = \delta_{mn}$ in infinite well

($I_{hm}^{en} \approx \delta_{mn}$ in finite potential well)



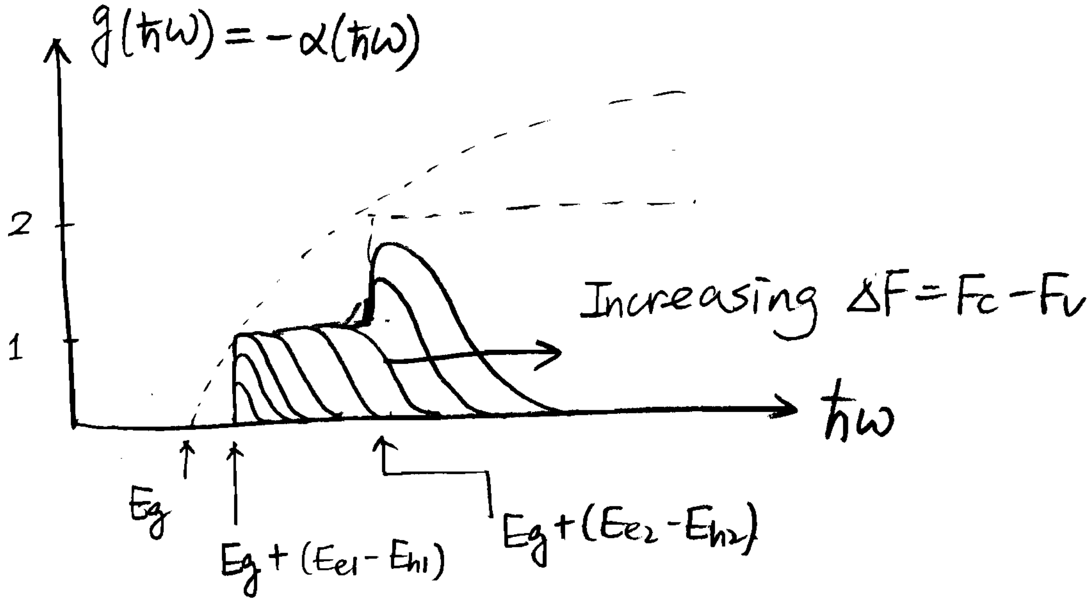
For a given photon energy ($\hbar\omega$), E_a and E_b are independent of $\vec{k}_t$

$$f_v(E_a) - f_c(E_b) = -\underbrace{f_g(\hbar\omega)}_{\text{Fermi-Inversion factor}} \text{ is only a function of } \hbar\omega$$

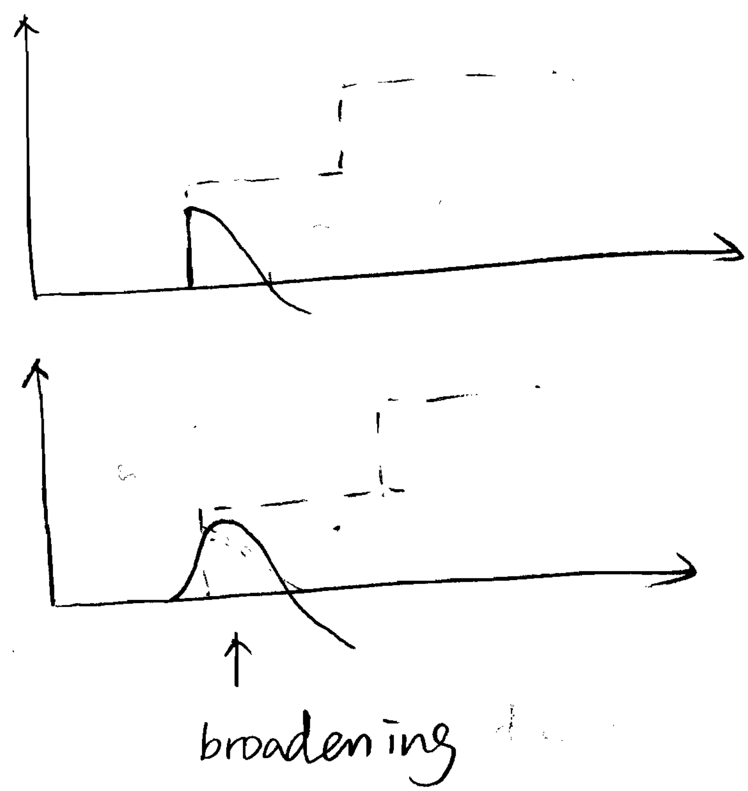


$$\Rightarrow \alpha(\hbar\omega) = \alpha_0(\hbar\omega)[-f_g(\hbar\omega)]$$

For symmetric QW, $I_{hm}^{en} = \delta_{mn}$



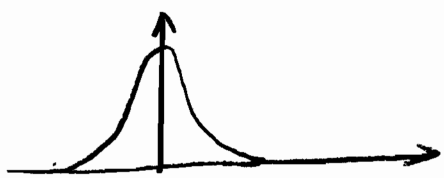
Compare with experiments



$\delta(E_t + E_{me}^{en} - \hbar\omega)$: zero linewidth

$$\rightarrow \frac{\Gamma/2\pi}{(E_t + E_{me}^{en} - \hbar\omega)^2 + (\Gamma/2)^2}$$ Finite linewidth, Γ

↳ Lineshape function $g(\nu)$



$\int_{-\infty}^{\infty} g(\nu)\, d\nu = 1$

$g(\nu)$ usually Lorentzian

$$\int \rho_r(E) \cdot \delta(E + E_g - \hbar\omega) \cdot (f_v - f_c)\, dE$$

↑
2D, or 3D

$$\rightarrow \int \rho_r(E) \cdot g(E)\, (f_v - f_c)\, dE$$

Quasi-Fermi levels in QW.

$$\underset{\substack{\uparrow \\ \text{electron} \\ \text{conc.}}}{N} = \sum_n \int dE \cdot \underbrace{\rho_e^{2D}}_{\frac{m_e^*}{\pi \hbar^2 \cdot L_z}}(E) \cdot f_c^n(E)$$

$$f_c^n = \frac{1}{e^{(E_{en} + E_t - F_c)/k_B T} + 1} \quad , \quad E_t = \frac{\hbar^2 k^2}{2 m_e^*}$$

$$\int \frac{dx}{1+e^x} = -\ln(1+e^{-x})$$

$$N = \sum_n \frac{m_e^* k_B T}{\pi \hbar^2 \cdot L_z} \cdot \ln\left(1 + e^{(F_n - E_{en})/k_B T}\right)$$

① $F_n \gg E_{en}$

$$\ln\left(1+e^{(F_n - E_{en})/k_B T}\right) \approx (F_n - E_{en}) \frac{1}{k_B T}$$

② $F_n \ll E_{en}$

$$\ln\left(1+e^{(F_n - E_{en})/k_B T}\right) \approx e^{-(E_{en} - F_n)/k_B T}$$

$$* \ \ln(1+\epsilon) \approx \epsilon$$

Most contributions of N are from energy levels below F_n