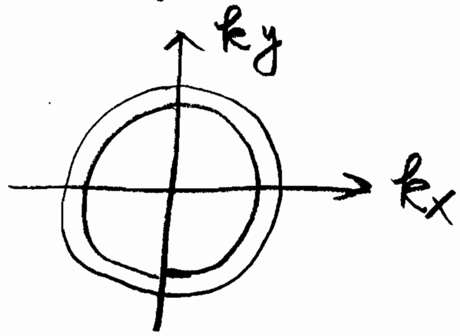
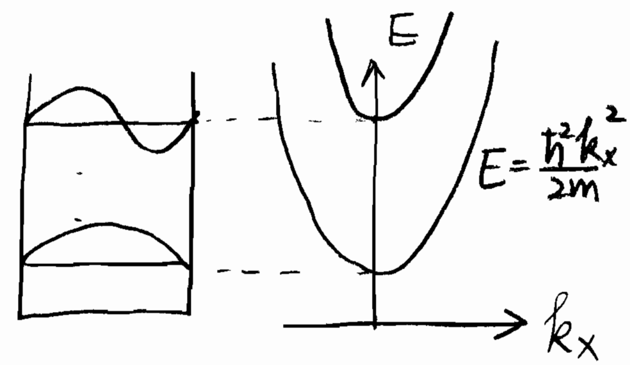
2-D Density of States

k_z is quantized





When $n=1$

Number of states from $k \to k + \Delta k$

$$\frac{2}{V} \cdot \int \frac{2\pi k \, dk}{\left(\frac{2\pi}{L_x}\right)\left(\frac{2\pi}{L_y}\right)} = \frac{2}{L_z} \cdot \frac{1}{2\pi} \cdot \int k \, dk$$

$$E_1 = \frac{\hbar^2}{2m^*}\left(k^2 + \left(\frac{\pi}{L}\right)^2\right)$$

$$dE = \frac{\hbar^2}{2m^*} \cdot 2k \, dk$$

$$= \frac{1}{\pi L_z} \cdot \frac{m^*}{\hbar^2} \int dE$$

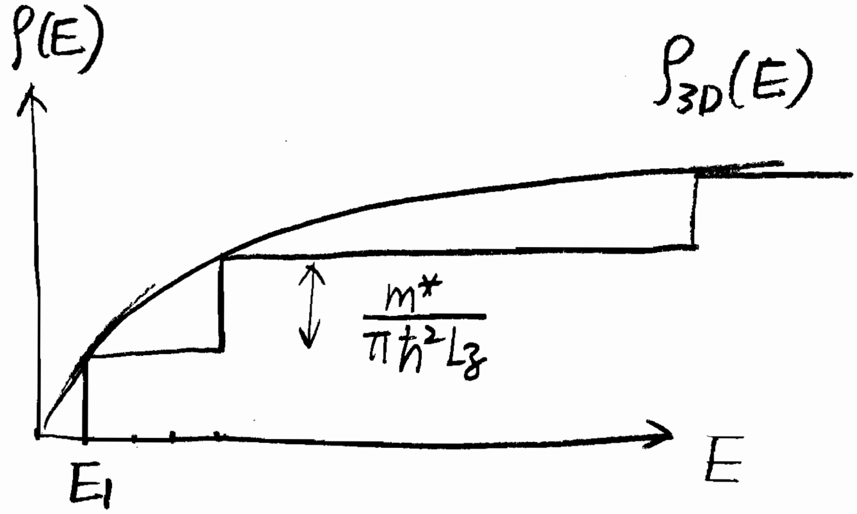
$E_1 < E < E_2$, only $n=1$ band $\quad \rho_{2D}(E) = \dfrac{m^*}{\pi \hbar^2 L_z}$

$E_2 < E < E_3$ $\quad n=1$ & $n=2$ $\quad \rho_{2D}(E) = \dfrac{m^*}{\pi \hbar^2 L_z} \times 2$

In general

$$\rho_{2D}(E) = \frac{m^*}{\pi \hbar^2 L_z} \cdot \sum_{n=1}^{\infty} H(E - E_n)$$

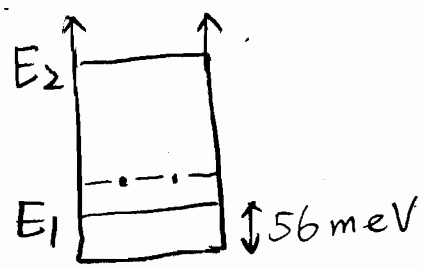
H ← step function



$\rho_{3D}(E) \propto \sqrt{E}$

$$n = \int \rho_{2D}(E) \cdot f(E) \cdot dE$$

Example: Electron concentration in a 10-nm quantum well in GaAs when Fermi energy is 30 meV above E_1



Uses infinite barrier approximation

$T = 0$

$$n = (E_F - E_1) \cdot \rho_{2D}(E_1 < E < E_2)$$

$$= (100\,\text{meV})\left(\frac{m_e^*}{\pi \hbar^2 \cdot L_z}\right)$$

$$56\,\text{meV} = \frac{\hbar^2}{2m_e^*} \cdot \frac{\pi^2}{L_z^2}$$

$$= \frac{\pi \hbar^2 L_z}{m_e^*} \cdot \frac{1}{2\cancel{\pi}\, L_z} \cdot \frac{\pi^2}{L_z^2}$$

$$\Rightarrow \frac{m_e^*}{\pi^2 \hbar^2 L_z} = \frac{1}{56\,\text{meV}} \cdot \frac{\pi}{2} \cdot \frac{1}{L_z^3}$$

$$\Rightarrow n = \frac{100}{56} \cdot \frac{\pi}{2} \cdot \left(\frac{1}{10^{-8}}\right)^3 \ \frac{1}{m^3}$$

$$= 2.8 \times 10^{24} \ \frac{1}{m^3}$$

$$= 2.8 \times 10^{18} \ 1/cm^3$$

$$n_{2D} = n \cdot L_z = 2.8 \times 10^{12} \ 1/cm^2$$