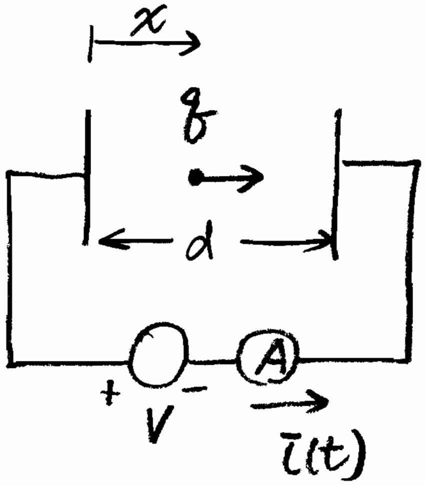


Current caused in the external circuit by a moving charge q in a parallel plate capacitor with a separation of d and voltage bias of V:

$$i(t) = \frac{q\,v(t)}{d}$$

Proof: work done on the charge

$$W = \text{Force} \cdot \text{distance} = qE \cdot dx = q\frac{V}{d} \cdot dx$$

work provided by power supply

$$W = V \cdot i(t) \cdot dt$$

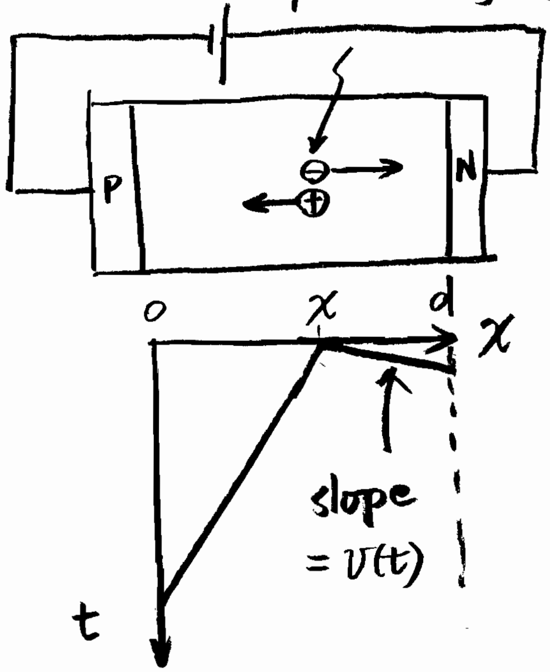
$$\Rightarrow V \cdot i(t)\,dt = V \cdot \frac{q}{d}\,dx$$

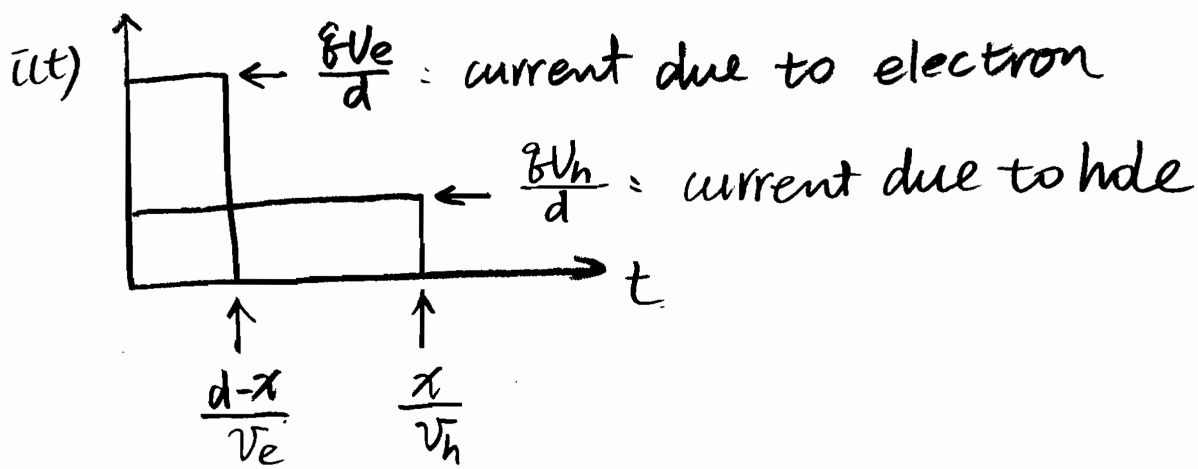
$$\Rightarrow i(t) = \frac{q}{d} \cdot \frac{dx}{dt} = \frac{q\,v(t)}{d}$$

#

Transit Time Concept

* Due to absorption of a single photon





Area = total charge induced in external circuit

$$\frac{qv_e}{d}\cdot\frac{d-x}{v_e} + \frac{qv_h}{d}\cdot\frac{x}{v_h} = q\left(\frac{d-x}{d}+\frac{x}{d}\right) = q$$

* Simple estimation of transit time

$\frac{d}{v_h}$ (hole is usually slower)

* Small-signal approach.

Incident optical field

$$e = E_s(1 + m \cdot \cos W_m t) \cdot \cos \omega t$$

↑ modulating freq (at W_m)   ↑ optical freq. (at ω)

$$= Re[V(t)]$$

$$V(t) = E_s(1 + m \cos W_m t) \cdot e^{i\omega t}$$

Generation rate

$$G(t) = a \langle \tfrac{1}{2} V(t) V^*(t) \rangle$$

↑ time average

$$= a \cdot E_s^2 \left[1 + \frac{m^2}{2} + 2m \, e^{iW_m t} \right]$$

$$i(t) = \frac{e\bar{v}}{d} \qquad \tau_d = \frac{d}{\bar{v}} \text{ : transit time}$$

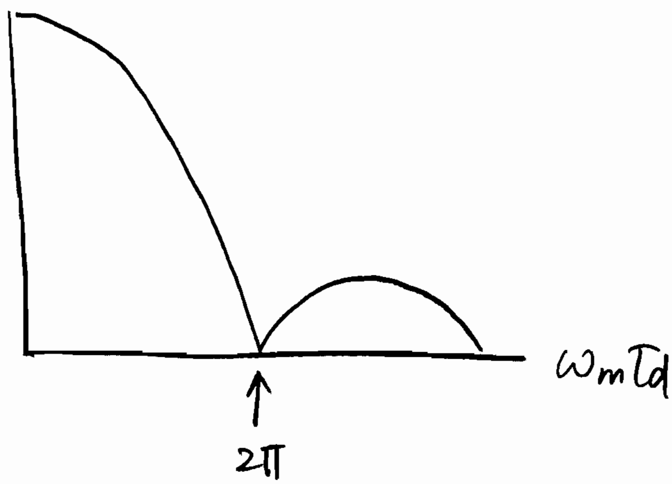
$$i(t) = \frac{e\bar{v}}{d} \cdot \int_{t-\tau_d}^{t} G(t')\,dt'$$

$$= \left(1 + \frac{m^2}{2}\right) e \cdot a \cdot E_s^2 + 2m \cdot e \cdot a \, E_s^2 \cdot \left(\frac{1 - e^{-iW_m \tau_d}}{iW_m \tau_d} \right) \cdot e^{iW_m t}$$

Setting $m = 0$ $\quad I = \dfrac{Pe\eta}{\hbar\omega} = e \cdot a \cdot E_s^2$

$$i(t) = \frac{P \cdot e \cdot \eta}{\hbar\omega} \left[\left(1 + \frac{m^2}{2}\right) + 2m \cdot \left(\frac{1 - e^{-iW_m \tau_d}}{iW_m \tau_d} \right) e^{iW_m t} \right]$$

$$\left| i(t)_{ac} \right| \propto \left| \frac{1-e^{-i\omega_m \tau_d}}{i\omega_m \tau_d} \right| = \frac{\sin\left(\frac{\omega_m \tau_d}{2}\right)}{\left(\frac{\omega_m \tau_d}{2}\right)}$$



3-dB point ($\frac{1}{\sqrt{2}}$)

$$\frac{\omega_m \tau_d}{2} \approx 1.4$$

$$2\pi f_m \tau_d \approx 2.8$$

$$f_m = \frac{2.8}{2\pi} \cdot \frac{1}{\tau_d} \approx 0.44 \frac{1}{\tau_d}$$

1/e point

$$\frac{\omega_m \tau_d}{2} \approx 2.2$$

$$f_m \approx \frac{4.4}{2\pi} \frac{1}{\tau_d}$$

Total response time

$$\tau = \tau_{RC} + \tau_t = RC + \frac{1}{4.4}\tau_d$$

$$f = \frac{1}{2\pi}\frac{1}{\tau}$$

Example

p-i-n . 10 μm × 10 μm ,

$R = 50\,\Omega$

$$RC = \frac{R\epsilon A}{d} = \frac{10^{-12} \cdot 10^{2} \times 10^{-8}}{d} \times 50$$

$$\tau_t = \frac{1}{4.4} \cdot \frac{d}{v_h}$$

$$RC + \tau_t \geq 2\sqrt{\frac{\epsilon A R}{d} \frac{d}{4.4 v_h}} = 2\sqrt{\frac{\epsilon A R}{4.4 v_h}}$$

Max freq achieved (min τ) when

$$RC = \tau_t$$

or $d = \sqrt{4.4\, v_h \cdot \epsilon A \cdot R}$ ← $50\,\Omega$

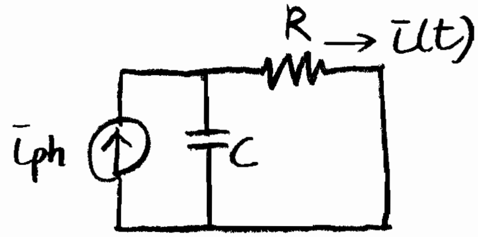
v_h: 5×10^{6} cm/sec, ϵ: 10^{-12} F/cm, A: 10^{-6} cm^2

$d \approx 0.3\,\mu m$

* More rigorous analysis

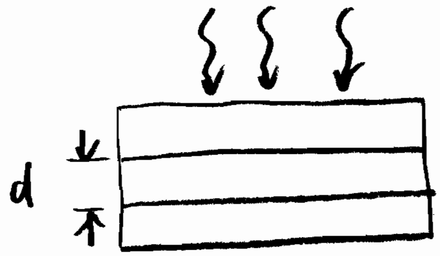
$$|i(t)| \propto \left| \frac{\frac{1}{i\omega C}}{R + \frac{1}{i\omega C}} \right| \cdot \text{sinc}\left(\frac{\omega_m \tau_d}{2}\right)$$

$$= \left| \frac{1}{1 + i\omega RC} \right| \cdot \text{sinc}\left(\frac{\omega_m \tau_d}{2}\right)$$



Equivalent circuit

Efficiency Consideration



Surface-illuminated p-i-n

$\eta = \eta_i (1 - e^{-\alpha d})$

There is a quantum efficiency — bandwidth trade-off

* High η → large d → long transit time
 → low bandwidth

* Simplified case, assume
  - Low efficiency $\eta(d) \approx \eta_i (1 - 1 + \alpha d) = \eta_i \alpha d$
  - Transit time limited BW

$$f_{3dB} \approx 0.44 \frac{v_h}{d}$$

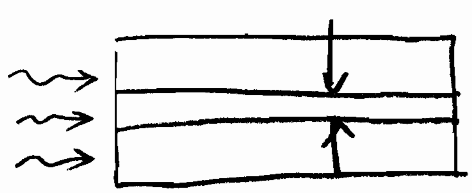
$$\eta \times f_{3dB} \approx \eta_i \alpha d \times 0.44 \frac{v_h}{d} \approx 0.44 \eta_i \alpha \cdot v_h$$

$\alpha \sim 10^4\ cm^{-1}$

$v_h \sim 5 \times 10^6\ cm/sec$

$\eta_i \sim 100\%$

$$\eta \times f_{3dB} \approx 2.2 \times 10^{10}$$



Waveguide PD (edge-illuminated) does not have $\eta - f_{3dB}$ trade-off