Dynamic Response of Semiconductor Lasers

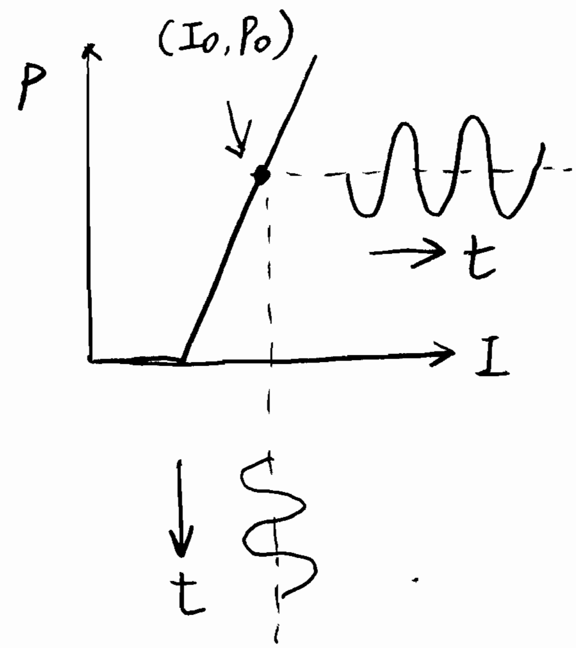
(Chap. 11)

Rate Eqs.

$$\begin{cases} \dfrac{dN}{dt} = \eta_i \dfrac{J}{qd} - \dfrac{N}{\tau} - v_g\, g(N)\cdot S \\[2ex] \dfrac{dS}{dt} = \Gamma v_g \cdot g(N)\cdot S - \dfrac{S}{\tau_p} + \beta R_{sp} \end{cases} \qquad v_g = \frac{c}{n_r}$$

Previously. we have solved steady state solution by setting $\frac{dN}{dt}=0$, $\frac{dS}{dt}=0$

$\Rightarrow$ N_0, S_0 with current density $J_0 \rightarrow$ Bias point.



$$P = \frac{\hbar\omega}{q}\eta_i \frac{\alpha_m}{\alpha_m+\alpha_i}(I - I_{th})$$

$$P = S\cdot \hbar\omega \frac{WdL}{\Gamma}\cdot v_g \cdot \alpha_m$$

$$S_0 = \frac{\beta R_{sp}}{\frac{1}{\tau_p} - \Gamma v g_0}$$

$$N_0 = \tau\left(\frac{J_0}{qd} - v g_0 S_0\right)$$

Small-signal analysis

$$J(t) = J_0 + j(t) = J_0 + Re[j(\omega) e^{-i\omega t}]$$
$$N(t) = N_0 + n(t) = N_0 + Re[n(\omega) e^{-i\omega t}]$$
$$S(t) = S_0 + s(t) = S_0 + Re[s(\omega) e^{-i\omega t}]$$

Taylor expansion of $g(N)$

$$g(N) = g(N_0) + \frac{dg}{dN}(N - N_0) = g_0 + g'(N - N_0)$$

Plug in Rate Eqs, keeping first-order terms ($e^{-i\omega t}$)

$$\begin{cases} \dfrac{dn(t)}{dt} = \eta_i \dfrac{j(t)}{qd} - \dfrac{n(t)}{\tau} - v_g \cdot [g' S_0 \cdot n(t) + g_0 s(t)] \\ \dfrac{ds(t)}{dt} = \Gamma \cdot v_g [g' S_0 n(t) + g_0 \cdot s(t)] - \dfrac{s(t)}{\tau_p} \end{cases}$$

$$\frac{d}{dt} \to -i\omega \cdot$$

$$\Rightarrow \begin{cases} -i\omega n(\omega) = \eta_i \dfrac{j(\omega)}{qd} - \dfrac{n(\omega)}{\tau} - v_g [g' S_0 \cdot n(\omega) + g_0 s(\omega)] \\ -i\omega s(\omega) = \Gamma v_g \cdot [g' S_0 n(\omega) + \cancel{g_0 s(\omega)}] - \cancel{\dfrac{s(\omega)}{\tau_p}} \end{cases}$$

$$\frac{1}{\tau_p} \approx \Gamma v_g g_0$$

$$n(\omega) = \frac{-i\omega}{\Gamma v_g g' S_0} s(\omega)$$

$$s(\omega) = \frac{(\Gamma v_g g' S_0) \cdot \eta_i \dfrac{j(\omega)}{qd}}{-\omega^2 - i\omega\left(\frac{1}{\tau} + v g' S_0\right) + \dfrac{v_g g'}{\tau_p} S_0}$$

Denominator

$$D(\omega) = -\omega^2 - i\omega\gamma + \omega_r^2$$

$$\omega_r = \sqrt{v_g \cdot g' \frac{S_0}{\tau_p}}$$

$$f_r = \frac{1}{2\pi}\omega_r$$ relation oscillation

$$\gamma = \frac{1}{\tau} + v_g g' S_0$$ damping factor

$$\frac{S(\omega)}{j(\omega)} = \underbrace{\frac{\eta_i}{qd} \cdot \frac{\Gamma v_g \cdot g' S_0}{\omega_r^2}}_{\text{Low freq response}} \cdot H(\omega)$$

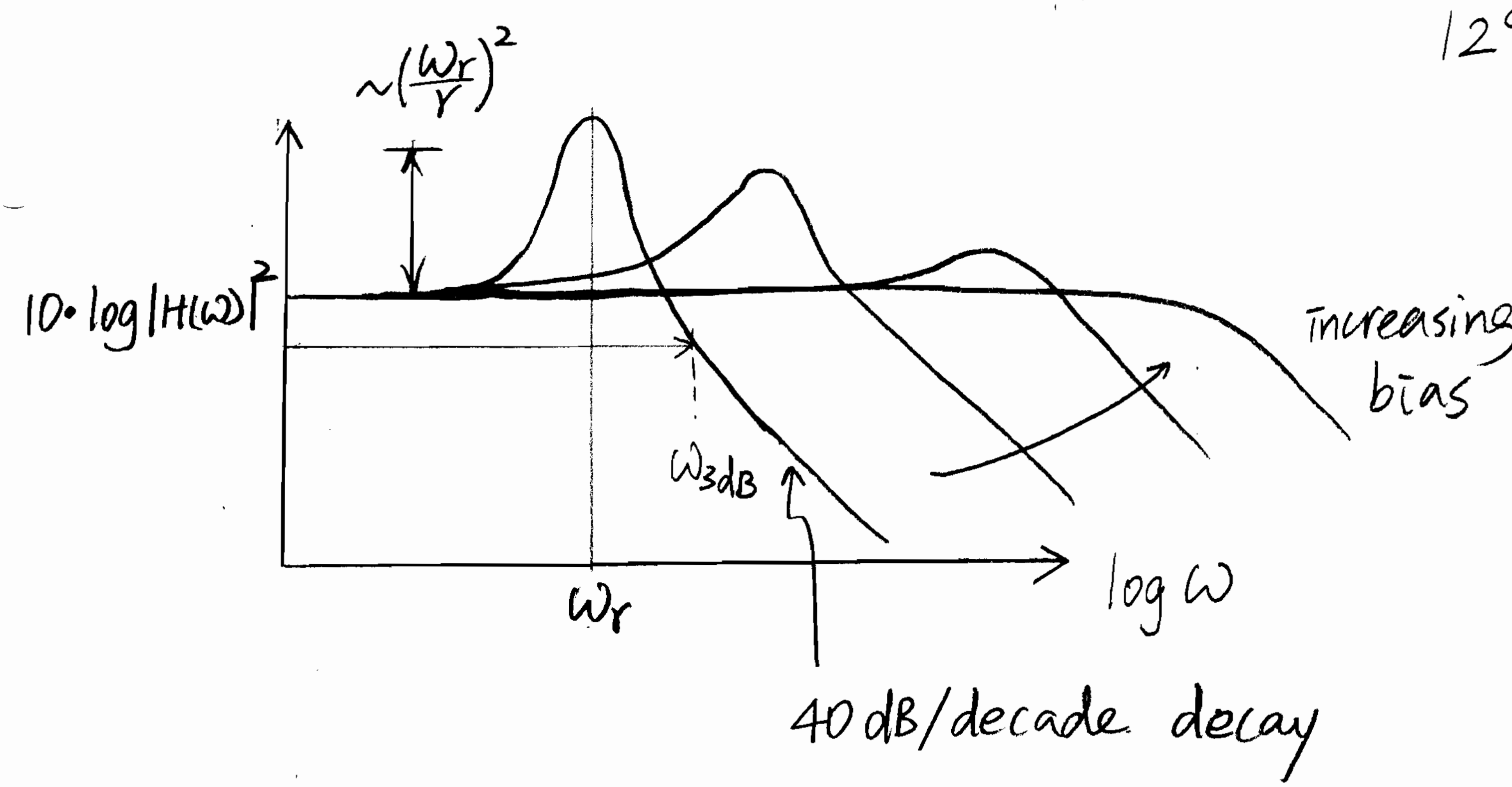
$$H(\omega) = \frac{\omega_r^2}{\omega_r^2 - \omega^2 - i\omega\gamma}$$

$$|H(\omega)|^2 = \frac{\omega_r^4}{[(\omega_r^2 - \omega^2)^2 + \omega^2\gamma^2]}$$

① $\omega \to 0$. $|H(\omega)|^2 \to 1$

② $\omega = \omega_r$ $|H(\omega_r)|^2 = \left(\frac{\omega_r}{\gamma}\right)^2$

③ $\omega \gg \omega_r$ $|H(\omega_r)|^2 \to \frac{\omega_r^4}{\omega^4}$



Note

$$\omega_r = \sqrt{v_g \cdot g' \frac{S_0}{\tau_p}} \propto \sqrt{S_0} \propto \sqrt{P_0} \propto \sqrt{I - I_{th}}$$

(P_0: output power)

Typical $f_r = \frac{1}{2\pi}\omega_r$

$v_g \sim 10^{10}$ cm/sec

$g' \sim 10^{-16}$ cm^2

$$P_o = \hbar\omega \cdot S_0 \cdot \frac{V_{active}}{\Gamma} \cdot \alpha_m \cdot v_g = 10\,\text{mW} = 10^{-2}$$

$\hbar\omega \to 1\,\text{eV}$, $\frac{V_{active}}{\Gamma} \to \frac{200\,\mu m^3}{0.5}$, $\alpha_m \to 50\ \text{cm}^{-1}$, $v_g \to 10^{10}\ \frac{\text{cm}}{\text{sec}}$

$S_0 \approx 4\times10^{14}\ 1/\text{cm}^3$

$$f_r \approx \frac{1}{2\pi}\sqrt{\frac{10^{10}\cdot 10^{-16}\cdot 4\times10^{14}}{10^{-12}}} \approx \frac{2\times10^{10}}{2\pi} \approx 3\ \text{GHz}$$

3-dB bandwidth

Neglect damping

$$|H(\omega)|^2 \sim \left|\frac{1}{1-\left(\frac{\omega}{\omega_r}\right)^2}\right|^2 \underset{\substack{\uparrow \\ \text{at } f_{3\text{-dB}}}}{=} \frac{1}{2}$$

$\omega_{3dB} > \omega_r$

$$\Rightarrow \quad \frac{1}{\left(\frac{\omega_{3dB}}{\omega_r}\right)^2 - 1} = \frac{1}{\sqrt{2}}$$

$$\left(\frac{\omega_{3dB}}{\omega_r}\right)^2 = 1+\sqrt{2}$$

$$\omega_{3dB} = \omega_r \cdot \sqrt{1+\sqrt{2}} \approx 1.55\,\omega_r$$

Low Frequency Modulation Efficiency

$$\frac{S(\omega\to 0)}{J(\omega\to 0)} = \frac{\eta_i}{qd}\cdot\frac{\Gamma v_g g' S_0}{\omega_r^2} = \frac{\eta_i}{qd}\cdot\frac{\cancel{\Gamma v_g g' S_0}}{\cancel{v_g g' S_0}} \times \tau_p = \frac{\eta_i}{qd}\Gamma\,\tau_p$$

$$\tau_p = \frac{1}{\Gamma v_g g_0} = \frac{1}{\Gamma v_g}\cdot\frac{1}{\frac{\alpha_0+\alpha_m}{\Gamma}} = \frac{1}{v_g}\cdot\frac{1}{\alpha_0+\alpha_m}$$

$$P_0 = S_0 \cdot \hbar\omega\,\frac{V_a}{\Gamma}\cdot\alpha_m\cdot v_g$$

$$i(\omega) = J(\omega)\cdot W\cdot L$$

$$\frac{P_0(\omega\to 0)}{i(\omega\to 0)} = \hbar\omega\cdot\frac{\cancel{V_a}}{\cancel{\Gamma}}\cdot\alpha_m\cdot\cancel{v_g}\cdot\frac{1}{\cancel{WL}}\cdot\frac{\eta_i}{\cancel{d}\cdot q}\cdot\cancel{\Gamma}\cdot\frac{1}{\cancel{v_g}}\cdot\frac{1}{\alpha_0+\alpha_m}$$

$$= \frac{\hbar\omega}{q}\cdot\eta_i\cdot\frac{\alpha_m}{\alpha_0+\alpha_m} \longrightarrow \text{DC external efficiency}$$

in $\frac{W}{A}$ (or $\frac{mW}{mA}$)

Damping Coefficient

$$\gamma = \frac{1}{\tau} + v_g \cdot g' \cdot S_0$$

In the presence of "nonlinear gain"

$$g(N) \rightarrow g(N, S) = \frac{g_0 + g'(N - N_0)}{1 + \epsilon S} = \frac{g(N)}{1 + \epsilon S}$$

$\epsilon \sim 1.5 \times 10^{-17}\ cm^3$ (for InGaAs/GaAs QW)

gain suppression coef

(or gain compression coef)

$$\gamma \rightarrow \frac{1}{\tau} + v_g \cdot g' S_0 + \frac{\epsilon S_0}{\tau_p}$$

$\uparrow$

additional damping term
due to nonlinear gain

$$\Rightarrow \gamma = \frac{v_g g' S_0}{\tau_p}\left(\tau_p + \frac{\epsilon}{v_g g'}\right) + \frac{1}{\tau}$$

It's customary to express γ in terms of K factor

$$\gamma = K \cdot f_r^2 + \frac{1}{\tau}$$ unit of K: sec

$$K = 4\pi^2\left(\tau_p + \frac{\epsilon}{v_g g'}\right)$$

$\tau_p \sim 1\,ps. \approx 10^{-12}$ $\quad$ $K \sim 0.43$ nsec

$$\frac{\epsilon}{v_g g'} \sim \frac{10^{-17}}{10^{10} \cdot 10^{-16}} \approx 10^{-11}$$

$$|H(\omega)|^2 = \frac{\omega_r^4}{(\omega_r^2-\omega^2)^2+\omega^2\gamma^2}$$

Denominator

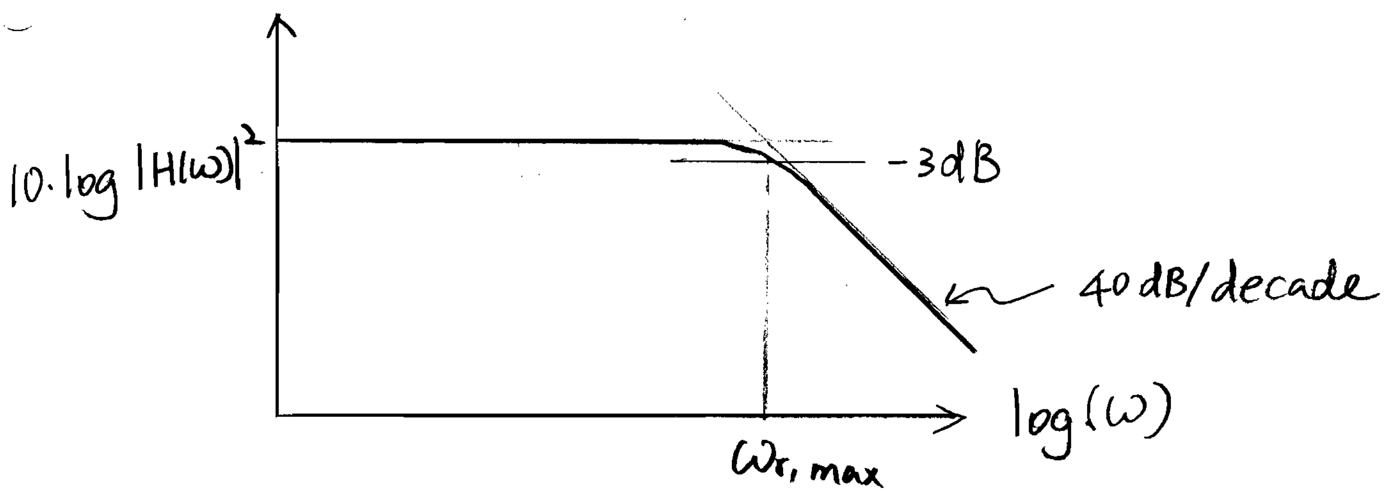
$$D(\omega) = \omega_r^4 \underline{- 2\omega_r^2\cdot\omega^2} + \omega^4 + \underline{\omega^2\cdot(k f_r^2)^2}$$

Maximum bandwidth occurs when

$$2\omega_r^2 = \gamma^2 = k^2\cdot f_r^4$$

$$2\cdot 4\pi^2 = k^2\cdot f_r^2$$

$$f_{r,max} = \frac{2\pi\sqrt{2}}{k} \approx \frac{8.9}{k}$$



$$|H(\omega)|^2 = \frac{\omega_{r,max}^4}{\omega^4+\omega_{r,max}^4}$$

$$|H(\omega_{r,max})|^2 = \frac{1}{2} \Rightarrow \omega_{3dB} = \omega_{r,max}$$