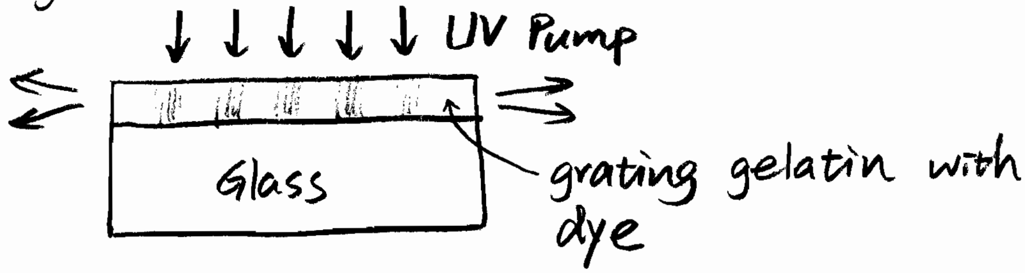
Distributed Feedback Laser (DFB)

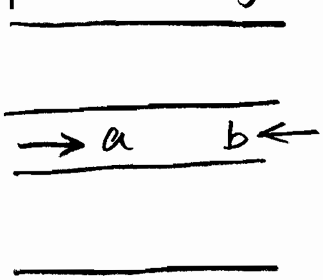
Kogelnik, Shank, 1971



Coupled Mode Theory

[Hermann A. Haus, Waves and Fields in Optoelectronics] 1984
Chap. 8.

Simple waveguide

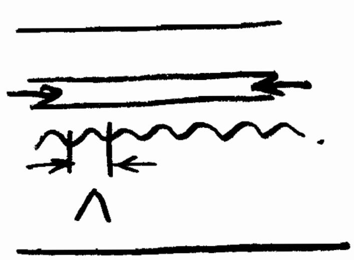


$$\frac{da}{dz} = i\beta a \quad \text{forward wave}$$

$$\frac{db}{dz} = -i\beta b \quad \text{backward wave}$$

WG with built-in grating

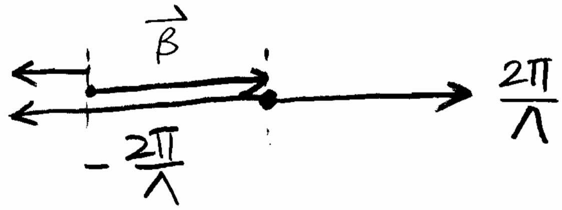
$$\Delta\epsilon(z) \sim \cos\frac{2\pi z}{\Lambda}$$



Forward wave

$$e^{+i\beta z}\cos\left(\frac{2\pi}{\Lambda}z\right) = \frac{1}{2}\left[e^{i(\beta+\frac{2\pi}{\Lambda})z} + e^{-i(\beta-\frac{2\pi}{\Lambda})z}\right]$$

Vector illustration



When $\beta - \frac{2\pi}{\Lambda} \approx -\beta \Rightarrow \beta = \frac{\pi}{\Lambda} = \frac{2\pi}{\lambda_g}$

or $\Lambda = \frac{\lambda_g}{2}$

the back scattered wave is coupled to the backward propagating wave.

Let the coupling constant be $j\kappa$

$$\frac{db}{dz} = -i\beta b + i\kappa_{ba}\, a \cdot e^{-j\frac{2\pi}{\Lambda}z}$$

Likewize

$$\frac{da}{dz} = i\beta a + i\kappa_{ab} \cdot b\, e^{+j\frac{2\pi}{\Lambda}z}$$

Define new variables

$$A(z) = a \cdot e^{-i\frac{\pi}{\Lambda}z} \sim e^{+i(\beta - \frac{\pi}{\Lambda})z} = e^{+i\Delta\beta}$$

$$B(z) = b\, e^{+i\frac{\pi}{\Lambda}z} \sim e^{-i(\beta - \frac{\pi}{\Lambda})z} = e^{-i\Delta\beta}$$

Note $|\beta| \sim \frac{\pi}{\Lambda} \Rightarrow |\Delta\beta| \equiv |\beta - \frac{\pi}{\Lambda}| \ll \beta$

$\Rightarrow$ $A(z)$ and $B(z)$ are "slowly varying envelops"

$$\frac{dA}{dz} = i\Delta\beta A + i\kappa_{ab} \cdot B$$

$$\frac{dB}{dz} = i\kappa_{ba} A - i\Delta\beta \cdot B$$

$$\frac{d}{dz}\begin{pmatrix} A \\ B \end{pmatrix} = i\begin{pmatrix} \Delta\beta & \kappa_{ab} \\ \kappa_{ba} & -\Delta\beta \end{pmatrix}\begin{pmatrix} A \\ B \end{pmatrix} = iq\begin{pmatrix} A \\ B \end{pmatrix}$$

$$\begin{pmatrix} A(z) \\ B(z) \end{pmatrix} = \begin{pmatrix} A_0 \\ B_0 \end{pmatrix} e^{iqz}$$

$$\Rightarrow \quad \begin{pmatrix} \Delta\beta - q & \kappa_{ab} \\ \kappa_{ba} & -\Delta\beta - q \end{pmatrix}\begin{pmatrix} A \\ B \end{pmatrix} = 0$$

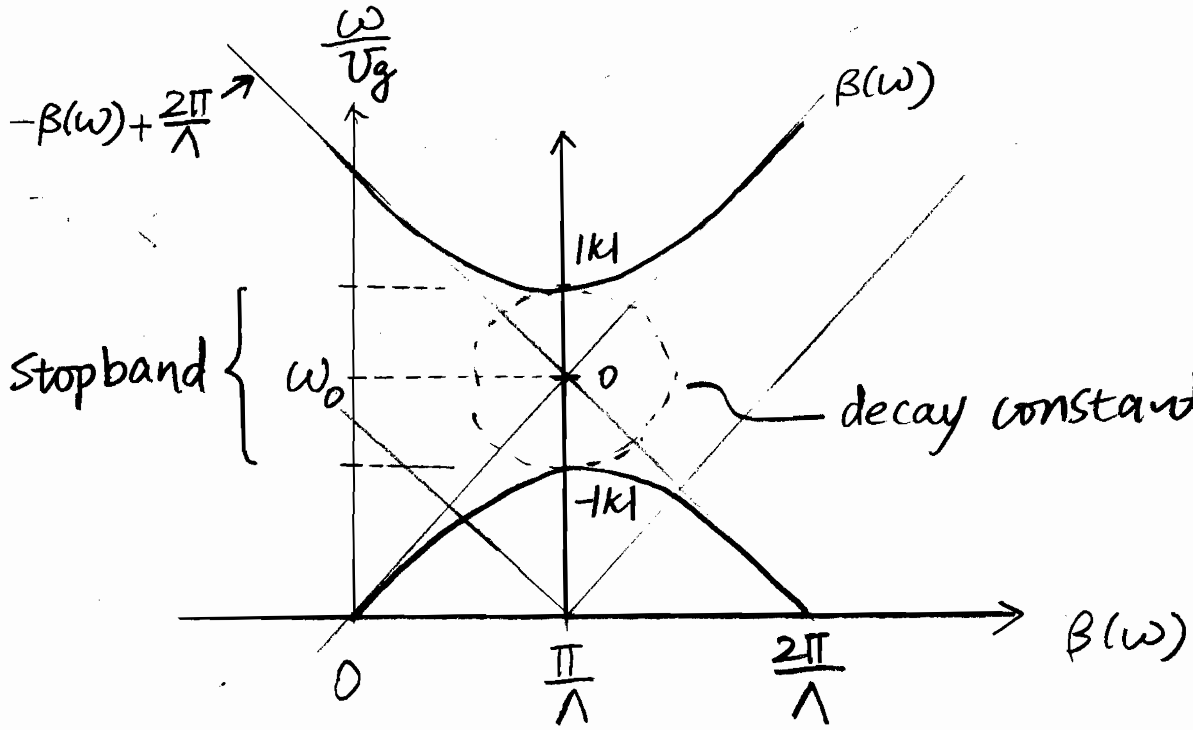
Lossless coupler $\quad \kappa_{ba} = -\kappa_{ab}^*$

Let $\kappa_{ab} = \kappa$, $\quad \kappa_{ba} = -\kappa^*$

$$\begin{vmatrix} \Delta\beta - q & \kappa \\ -\kappa^* & -\Delta\beta - q \end{vmatrix} = 0$$

$$q^2 - \Delta\beta^2 + |\kappa|^2 = 0$$

$$q = \pm\sqrt{\Delta\beta^2 - |\kappa|^2}$$



Expansion of $\beta(\omega)$ around $\omega_0 = 2\pi \frac{c}{\lambda_g} = 2\pi \frac{c}{(2\Lambda)}$

$$\beta(\omega) = \underbrace{\beta(\omega_0)}_{= \frac{\pi}{\Lambda}} + \underbrace{\frac{d\beta}{d\omega}}_{= \frac{1}{v_g}}(\omega-\omega_0)$$

$$\Delta\beta = \beta(\omega) - \frac{\pi}{\Lambda} = \frac{\omega-\omega_0}{v_g}$$

Asymptotic analysis

When $|\Delta\beta| \gg |\kappa|$

$$\gamma = \pm\sqrt{\Delta\beta^2 - |\kappa|^2} \approx \pm\Delta\beta$$

$$A(z) \sim e^{\pm i\Delta\beta \cdot z} = e^{\pm i(\beta - \frac{\pi}{\Lambda})z}$$

$$a(z) = A(z)\, e^{i\frac{\pi}{\Lambda}z} \sim \begin{cases} e^{+i\beta z} & \text{forward wave} \\ e^{-i\beta z + i\frac{2\pi}{\Lambda}z} & \text{reflected "} \end{cases}$$

When $|\Delta\beta| < |k|$

$$\gamma = \pm i S = \pm i \sqrt{|k|^2 - \Delta\beta^2}$$

$$e^{\pm i \gamma z} = e^{\mp S z} \quad \text{exponential decay}$$

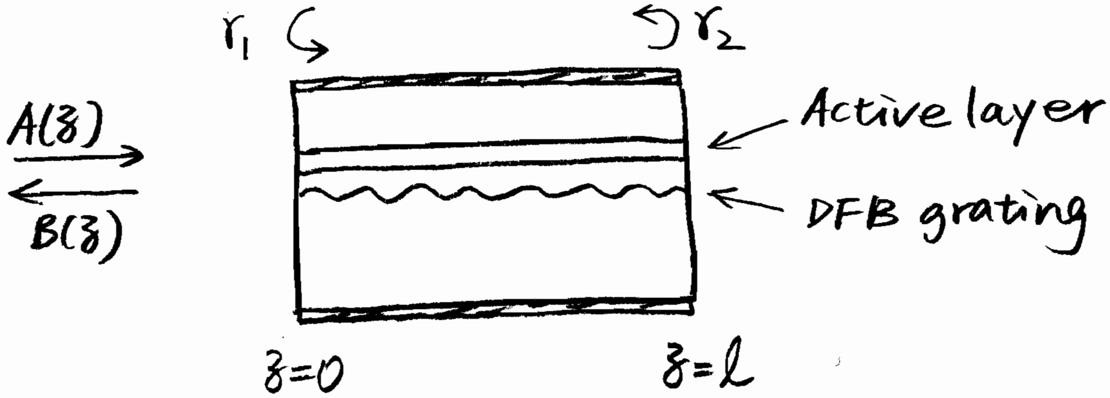
$\Rightarrow$ Stopband

$$\begin{pmatrix} A(z) \\ B(z) \end{pmatrix} = \begin{pmatrix} A_0 \\ B_0 \end{pmatrix} e^{\pm S z}$$

Eigenvector:

$$\begin{cases} \text{For } e^{-Sz}: & \dfrac{A_0}{B_0} = \dfrac{-k}{\Delta\beta - iS} \\[2ex] \text{For } e^{+Sz}: & \dfrac{A_0}{B_0} = \dfrac{-k}{\Delta\beta + iS} \end{cases}$$

DFB Laser (p.457 Chuang)



Propagation of the 2 eigen modes

$$\begin{bmatrix} A(z) \\ B(z) \end{bmatrix} = \begin{bmatrix} A^+ \\ B^+ \end{bmatrix} e^{iqz} + \begin{bmatrix} A^- \\ B^- \end{bmatrix} e^{-iqz}$$

$$\frac{B^+}{A^+} = \frac{\Delta\beta - q}{-\kappa} = \frac{-\kappa^*}{\Delta\beta + q}$$

$$\frac{B^-}{A^-} = \frac{\Delta\beta + q}{-\kappa} = \frac{-\kappa^*}{\Delta\beta - q}$$

"DFB reflection"

$$r_p(q) = \frac{B^+}{A^+} = \frac{-\kappa^*}{\Delta\beta + q} \quad \text{for the "+" } (e^{iqz}) \text{ mode}$$

$$r_m(q) = \frac{A^-}{B^-} = \frac{-\kappa}{\Delta\beta + q} \quad \text{for the "−" } (e^{-iqz}) \text{ mode}$$

Consider the simplest case, both facets are anti-reflection (AR)-coated such that

$$r_1 = 0$$

$$r_2 = 0$$

Boundary Conditions

At $z=0$. $A(0)=r_1 B(0)=0$

$$\Rightarrow A^+ + A^- = 0 \quad ------ \quad ①$$

At $z=l$. $B(l)=r_2 \cdot A(l)=0$

$$\Rightarrow B^+ e^{iql} + B^- e^{-iql} = 0$$

$$\begin{cases} B^+ = A^+ \dfrac{\Delta\beta - q}{-\kappa} = A^+ \dfrac{-\kappa^*}{\Delta\beta + q} \\ B^- = A^- \dfrac{\Delta\beta + q}{-\kappa} \end{cases}$$

$$\Rightarrow A^+ (\delta - \Delta\beta) e^{iql} + A^- (\delta + \Delta\beta) e^{-iql} = 0 \quad -- \quad ②$$

Nontrivial solution for ① & ②

$$\begin{vmatrix} 1 & 1 \\ (\Delta\beta - q) e^{iql} & (\Delta\beta + q) e^{-iql} \end{vmatrix} = 0$$

$$(\Delta\beta + q) e^{-iql} - (\Delta\beta - q) e^{iql} = 0$$

$$q(e^{iql} + e^{-iql}) = \Delta\beta (e^{iql} - e^{-iql})$$

$$q \cdot 2 \cdot \cos ql = \Delta\beta \cdot 2i \sin ql$$

$$\Rightarrow q = i\Delta\beta \tan(ql)$$

$\Rightarrow$ Threshold condition for DFB laser

But how do we incorporate $g(N)$?

Gain is the imaginary part of β

$$\beta \to \frac{2\pi}{\lambda} n - i\frac{\Gamma g}{2}$$

$$\Delta\beta = \beta - \frac{\pi}{\Lambda} \to \underbrace{\left(\frac{2\pi}{\lambda} n - \frac{\pi}{\Lambda}\right)}_{\delta} - i\frac{\Gamma g}{2}$$

$\delta \uparrow$ real number

$$\gamma = \sqrt{\left(\delta - i\frac{\Gamma g}{2}\right)^2 - |\kappa|^2}$$

Threshold condition

$$\sqrt{\left(\delta - i\frac{\Gamma g}{2}\right)^2 - |\kappa|^2} = i\cdot\left(\delta - i\frac{\Gamma g}{2}\right)\cdot \tan\left[\sqrt{\left(\delta - i\frac{\Gamma g}{2}\right) - |\kappa|^2}\cdot l\right]$$

Two equations (real, imaginary)

2 unknowns (δ, g)

$|\kappa|$ grating strength is a given parameter

These equations usually need to be solved numerically

$$\begin{cases} \text{solution of } \delta \longrightarrow \text{modes} \\ \text{solution of } g \longrightarrow \text{threshold gain} \end{cases}$$

Reflectivity of Passive Bragg grating:

Follow the same formulation,

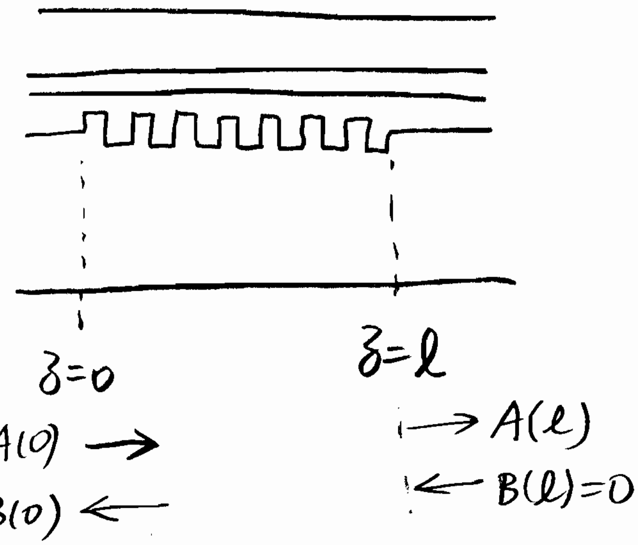
set $g = 0$

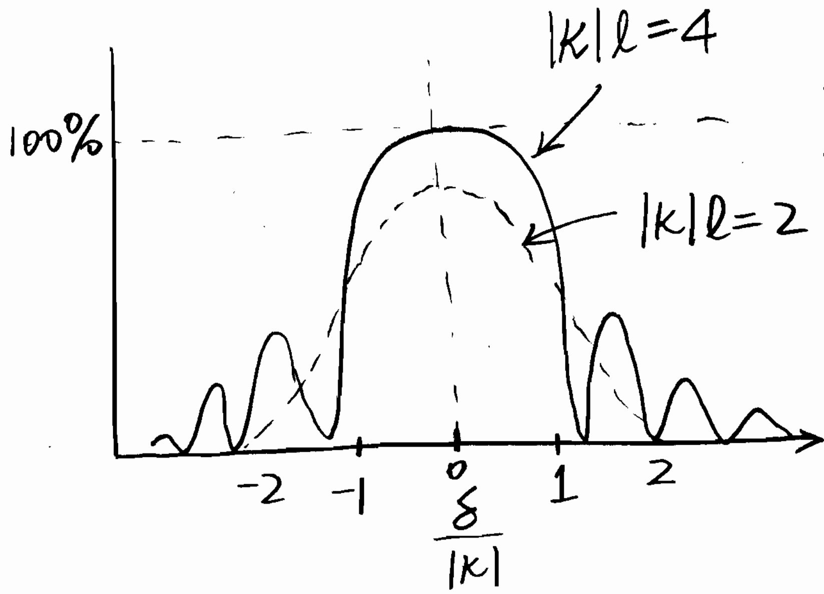
Use different boundary conditions:

$$B(l) = 0$$

$$r = \frac{B(0)}{A(0)}$$

$$R = |r|^2$$





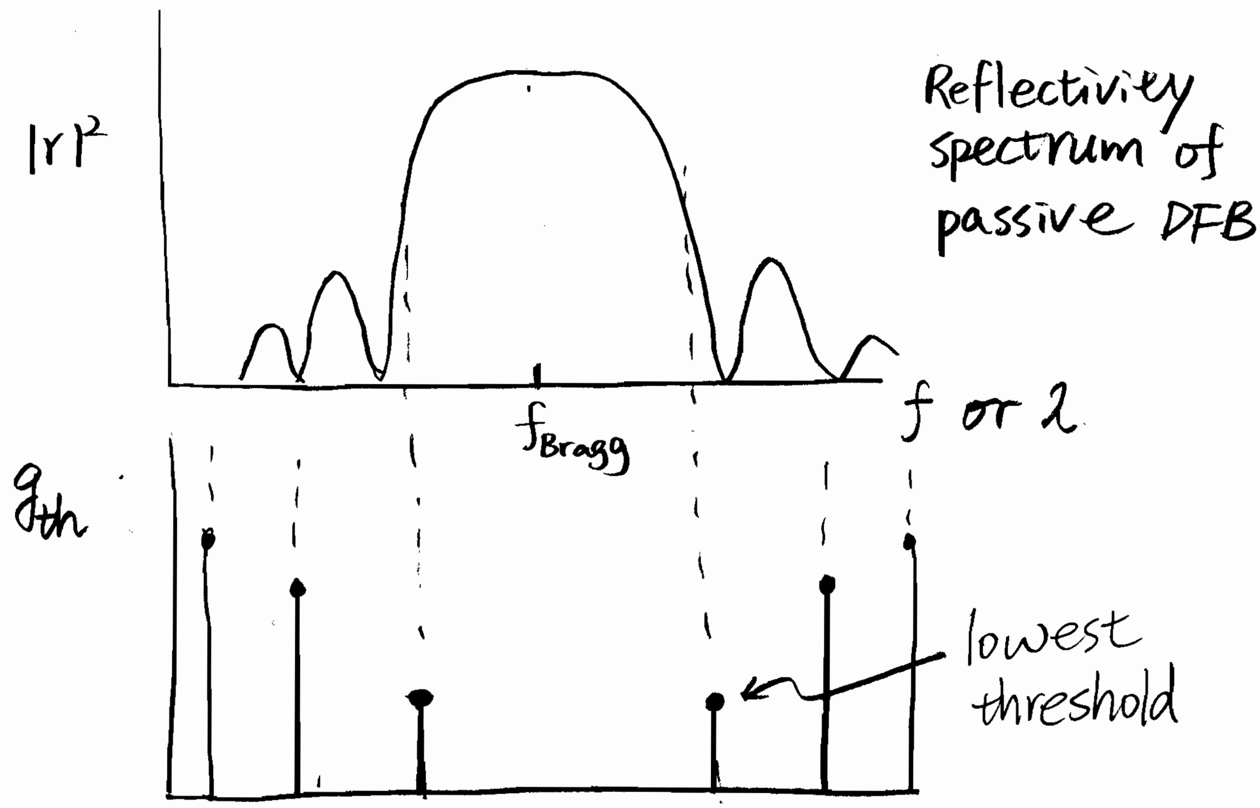
* Reflectivity can reach ~100% with sufficiently large $|\kappa| \cdot l$ (> 3)

* For large $|\kappa| l$, the reflection band (often called stopband) width

$$\sim 2 \cdot |\kappa| \quad (cm^{-1})$$

Half width: $d \cdot \delta = d \cdot \Delta\beta = d\beta = d\left(\frac{2\pi}{\lambda} n\right) = \frac{2\pi n}{\lambda^2} \cdot (-d\lambda) = |\kappa| \Rightarrow |d\lambda| = \frac{\lambda^2}{2\pi n} |\kappa|$

Solution of DFB threshold condition

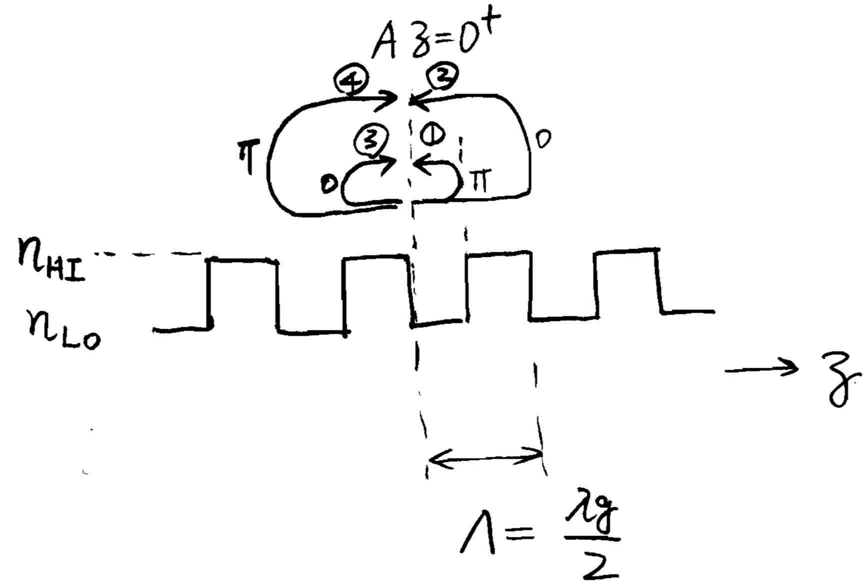


* DFB is frequency-selective.
  Modes closest to the Bragg wavelength ($\lambda_{Bragg} = 2 \cdot n \cdot \Lambda$) have lowest threshold.
  (In comparison, all modes in FP laser have the same threhold. The gain peak selects the peak λ, and usually there are multiple modes)

* Modes as determined by the threshold condition are NOT uniformly spaced.
  - No mode inside the stopband
  - lasing modes near the edges of the stopband

- There are 2 degenerate modes for DFB lasers with both facets AR-coated.

Simple intuitive picture why there is no modes in the stopband (high reflectivity region)



Phase shift for

reflection is negative when going from high to low index

① $2\beta\left(\frac{\Lambda}{2}\right) + \pi = \frac{2\pi}{\lambda_g} \cdot \frac{\lambda_g}{2} + \pi = 2\pi$

② $2\beta \cdot \Lambda + 0 = 2\pi + 0 = 2\pi$

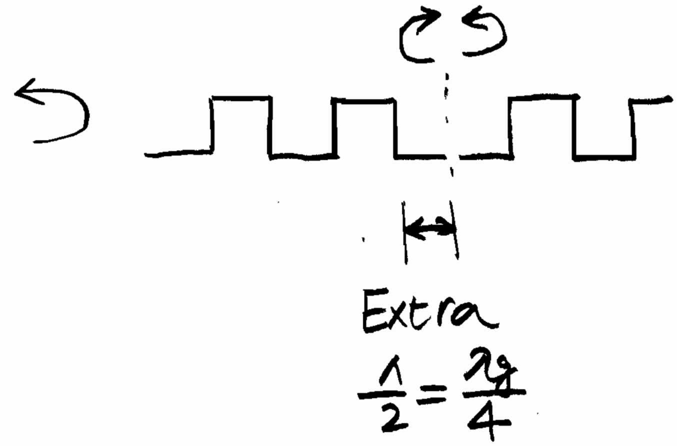
③ $2\beta\left(\frac{\Lambda}{2}\right) + 0 = \pi + 0 = \pi$

④ $2\beta(\Lambda) + \pi = 2\pi + \pi = 3\pi$

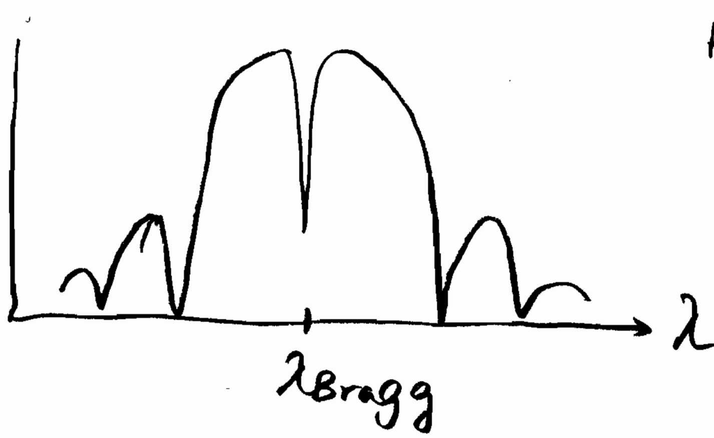
At Bragg wavelength, reflection from left and from right are out of phase

$\Rightarrow$ No mode at λ_{Bragg}

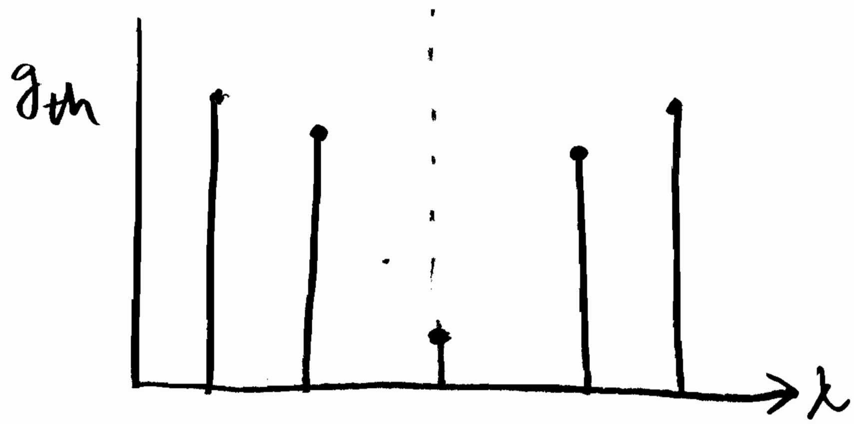
- Quarter-Wave Shifted DFB



Reflection from left and from right are now in phase



Reflectivity spectrum measured from outside



* lasing mode in the middle of stopband where reflectivity is highest.
* No degeneracy