Quantum Well Lasers

Advantages of QW lasers

- Low threshold current

$$I_{th} = \frac{N_{th}}{\tau_e} q \cdot d \cdot W \cdot L , \quad d \sim 10 \text{ nm for QW}$$

$$N_{th} = N_{tr} + \Delta N$$

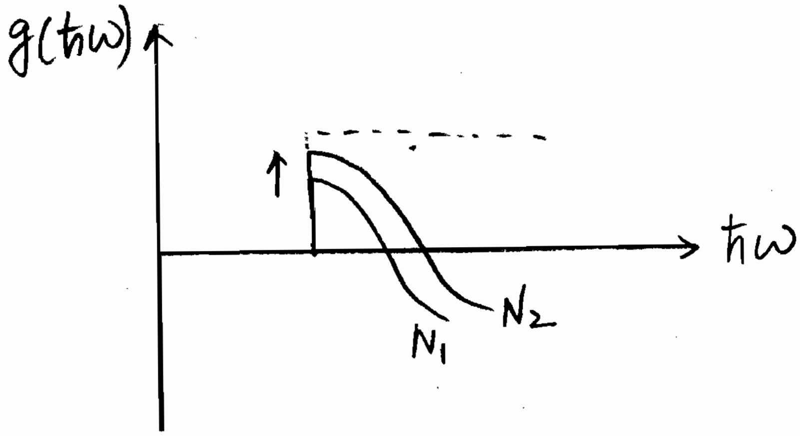
N_{tr}: Transparency carrier conc. (at which gain = 0)

ΔN: Additional carrier conc. to reach g_{th}

Both d and N_{tr} are smaller in QW laser

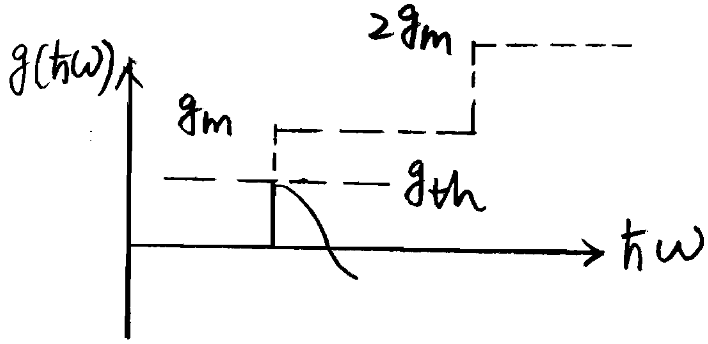
- Higher differential gain
  ⇒ Larger bandwidth
- Lower chirp
  i.e. smaller wavelength shift when the laser is modulated

Main reason: step-like density of states



* differential gain $= \dfrac{\Delta g_{max}}{\Delta N}$
* Peak $\hbar\omega \sim E^{el}_{h1}(0)$

For most of the benefits, we need to keep $g_{th} < g_m$



If $g_{th} > g_m$, we need to fill the first band until $\Delta E_F > E_{h2}^{e2}(0)$, resulting in high I_{th}

Consider first step

$$g_m = C_0 |\hat{e}\cdot\vec{M}|^2 |I_{h1}^{e1}|^2 \rho_r^{2D} \approx C_0 |\hat{e}\cdot\vec{M}|^2 \rho_r^{2D}$$

$$C_0 = \frac{\pi e^2}{n_r c \epsilon_0 m_0^2 \omega}$$

$$\rho_r^{2D} = \frac{m_r}{\pi \hbar^2 L_z}$$

$$g(\hbar\omega) = g_m \left(f_c(E_t = \hbar\omega - E_{h1}^{e1}) - f_v(E_t = \hbar\omega - E_{h1}^{e1}) \right)$$

Gain peak : $\hbar\omega \approx E_{h1}^{e1}$

$$g_p = g_m \left(f_c(E_t = 0) - f_v(E_t = 0) \right)$$

$$f_c(E_t = 0) = \frac{1}{1 + e^{(E_{e1} - F_c)/k_B T}} \qquad F_c > E_{e1}$$

$$= \frac{1}{1 + e^{-(F_c - E_{e1})/k_B T}}$$

$$\approx 1 - e^{-(F_c - E_{e1})/k_B T}$$

Electron Conc. (p.51 of Notes)

$$N = N_c \cdot \ln\left(1 + e^{(F_c - E_{e1})/k_B T}\right)$$

$$\approx N_c \cdot \frac{F_c - E_{e1}}{k_B T}$$

$$N_c = \frac{m_e^* k_B T}{\pi \hbar^2 L_z}$$

$$\Rightarrow \quad \frac{F_c - E_{e1}}{k_B T} \approx \frac{N}{N_c}$$

$$f_c(E_t = 0) \simeq 1 - e^{-\frac{N}{N_c}}$$

More accurate approximation (p.429 Chuang)

$$f_c(E_t = 0) = 1 - e^{-\frac{N}{N_c'}}$$

$$N_c' = N_c \cdot \sum_{n=1}^{\infty} e^{\frac{E_{e1} - E_{en}}{k_B T}}$$

$$f_v(E_t = 0) \approx e^{-\frac{P}{N_v}}$$

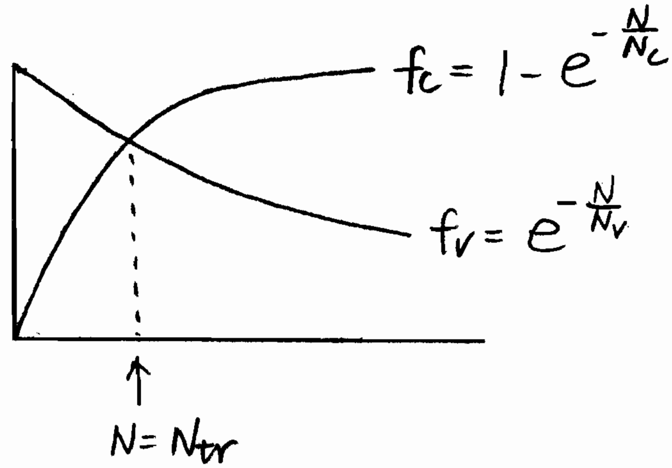
$$N_v = \frac{m_h^* k_B T}{\pi \hbar^2 L_z} \qquad N_v' = N_v \cdot \sum_{m=1}^{\infty} e^{\frac{E_{hm} - E_{h1}}{k_B T}}$$

Gain Peak

$$g_p = g_m \cdot \left(1 - e^{-\frac{N}{N_c}} - e^{-\frac{P}{N_v}}\right) \quad , \; N = P$$

Define $\frac{N_v}{N_c} = \frac{m_h^*}{m_e^*} \equiv R \quad (R \gg 1)$

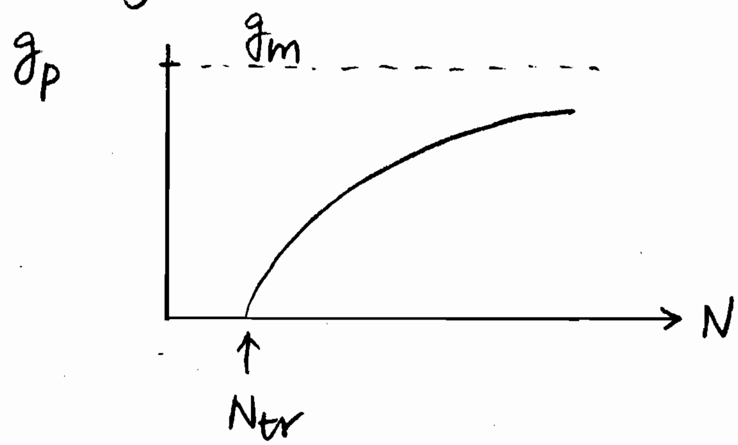
$$g_p = g_m \left(1 - e^{-\frac{N}{N_c}} - e^{-\frac{N}{RN_c}}\right)$$



Transparency carrier concentration N_{tr} : $g_p = 0$

$$e^{-\frac{N_{tr}}{N_c}} + e^{-\frac{N_{tr}}{RN_c}} = 1$$

Peak gain



The gain-vs-current curve can be very well by a logrithmatic shape:

$$g_p(N) = g_0 \ln \frac{N}{N_{tr}}$$

Alternative expression in Chuang

$$g_p(N) = g_0 \left(1 + \ln \frac{N}{N_0}\right)$$

$$N_0 = N_{tr} \cdot e^1$$

To avoid numerical singularity at $N=0$,
a 3-parameter gain curve is introduced.
[Ref: Coldren. P.167]

$$g = g_0 \ln\left(\frac{N+N_s}{N_{tr}+N_s}\right)$$

Note that g_0 in different fitting equations has different value.
N_s is a fitting parameter.

Example.

GaAs / $Al_{0.2}Ga_{0.8}As$ QW. $L_z = 8$ nm.

$$g = g_0 \ln \frac{N}{N_{tr}} \qquad \begin{cases} g_0 = 2400 \text{ cm}^{-1} \\ N_{tr} = 2.6\times10^{18} \text{ cm}^{-3} \end{cases}$$

$$g = g_0 \ln\left(\frac{N+N_s}{N_{tr}+N_s}\right) \qquad \begin{cases} g_0 = 3000 \text{ cm}^{-1} \\ N_{tr} = 2.6\times10^{18} \text{ cm}^{-3} \\ N_s = 1.1\times10^{18} \text{ cm}^{-3} \end{cases}$$

Differential Gain

$$\frac{dg_p}{dN} = \frac{g_0}{N} \quad \text{if} \quad g_p = g_0 \ln \frac{N}{N_{tr}}$$

$$\frac{dg_p}{dN} = \frac{g_0}{N+N_s} \quad \text{if} \quad g_p = g_0 \ln\left(\frac{N+N_s}{N_{tr}+N_s}\right)$$

<u>Gain - current density relation:</u>

$$J = A \cdot N + B \cdot N^2 + C N^3$$

$\downarrow$ Nonrad, $\downarrow$ spon, $\downarrow$ Auger

$$g_p(J) = g_{0J} \ln\left(\frac{J}{J_{tr}}\right)$$

$\uparrow$ different from g_0 discussed earlier

For the same example. (

8 nm $GaAs/Al_{0.2}Ga_{0.8}As$ QW

$$J_{tr} = 110 \ A/cm^2$$

$$g_{0J} = 1300 \ cm^{-1}$$

In other words.

$$\frac{J}{J_{tr}} = \left(\frac{N}{N_{tr}}\right)^{\frac{g_0}{g_{0J}}} = \left(\frac{N}{N_{tr}}\right)^{\beta}$$

β depends which recombination mechanism dominates

β is usually between $2 \sim 3$

Multiple Quantum Well (MQW)

Single QW works well when

$$g_{th} \ll g_m$$

When $g_{th} \sim g_m$ or $> g_m$, MQW is better

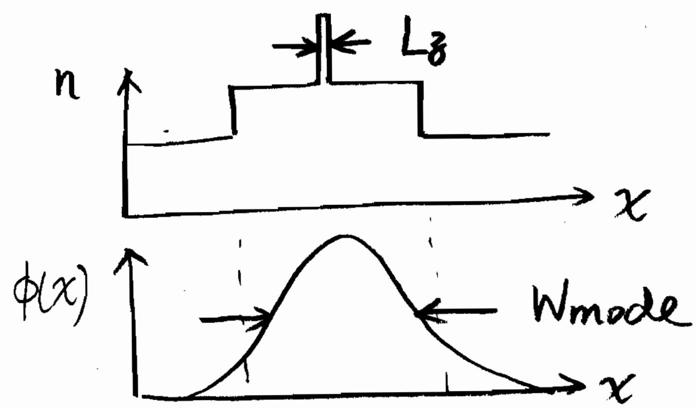
For MQW with n_w QWs,

$$G_{th} = n_w \Gamma_w \cdot g_w = \alpha_i + \alpha_m = \alpha_i + \frac{1}{2L} \ln \frac{1}{R_1 R_2}$$

g_w : material gain of QW

Γ_w : confinement factor per QW

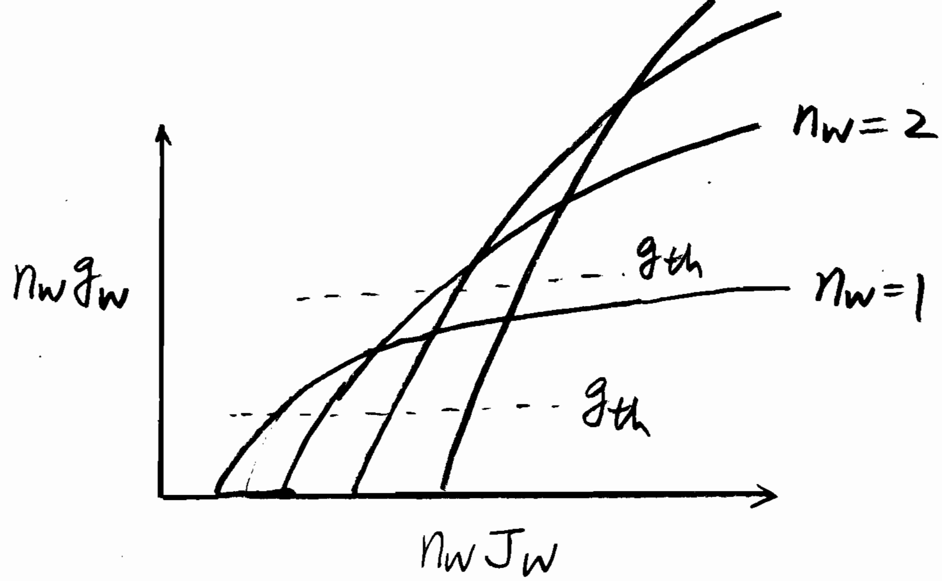
$$\Gamma_w = \frac{L_z}{W_{mode}}$$



Separate confinement Heterostructure (SCH)

$$g_w = g_m (f_c - f_v)$$

$\Gamma_w g_m$: maximum gain per QW



Depending on magnitude of g_{th}, there is an optimum number of QW for lowest I_{th}

Compare $n_w = 1$ and $n_w = 2$,

- $n_w = 2$ has $2\times$ J_{tr} ($g_w = 0$)
- $n_w = 2$ has $2\times$ max gain.

$$g_w = g_{0J} \ln\left(\frac{J_w}{J_{tr}}\right)$$

$$n_w g_w = n_w g_{0J} \cdot \ln\left(\frac{n_w J_w}{n_w J_{tr}}\right)$$

At threshold

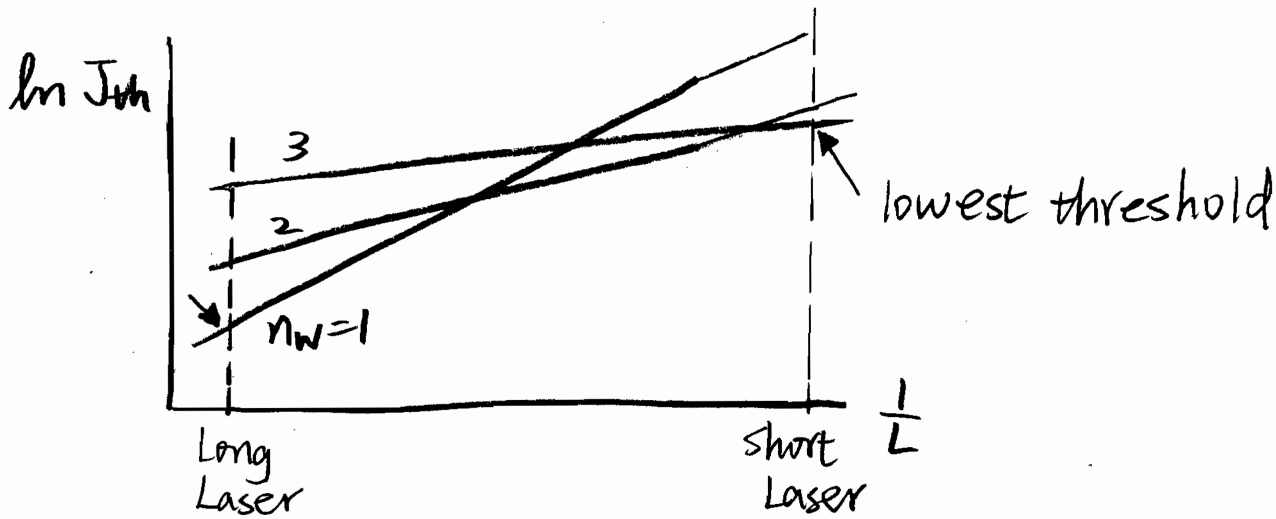
$$n_w g_w \Gamma_w = \alpha_i + \alpha_m$$

$$= n_w g_{0J} \cdot \Gamma_w \cdot \ln\left(\frac{J_w}{J_{tr}}\right)$$

$$J_w = J_{tr} \cdot \exp\left(\frac{\alpha_i + \alpha_m}{n_w g_{0J} \Gamma_w}\right)$$

$$J_{th} = \frac{n_w J_w}{\eta_i} = \frac{n_w J_{tr}}{\eta_i} \exp\left[\frac{1}{n_w g_{0J} \Gamma_w}\left(\alpha_i + \frac{1}{2L} \ln \frac{1}{R_1 R_2}\right)\right]$$

$$\ln J_{th} = \ln \frac{n_w J_{tr}}{\eta_i} + \frac{\alpha_i}{n_w g_{oJ} \Gamma_w} + \frac{1}{n_w g_{oJ} \Gamma_w} \ln \frac{1}{R_1 R_2} \cdot \frac{1}{2L}$$



$$I_{th} = W \cdot L \cdot J_{th}$$