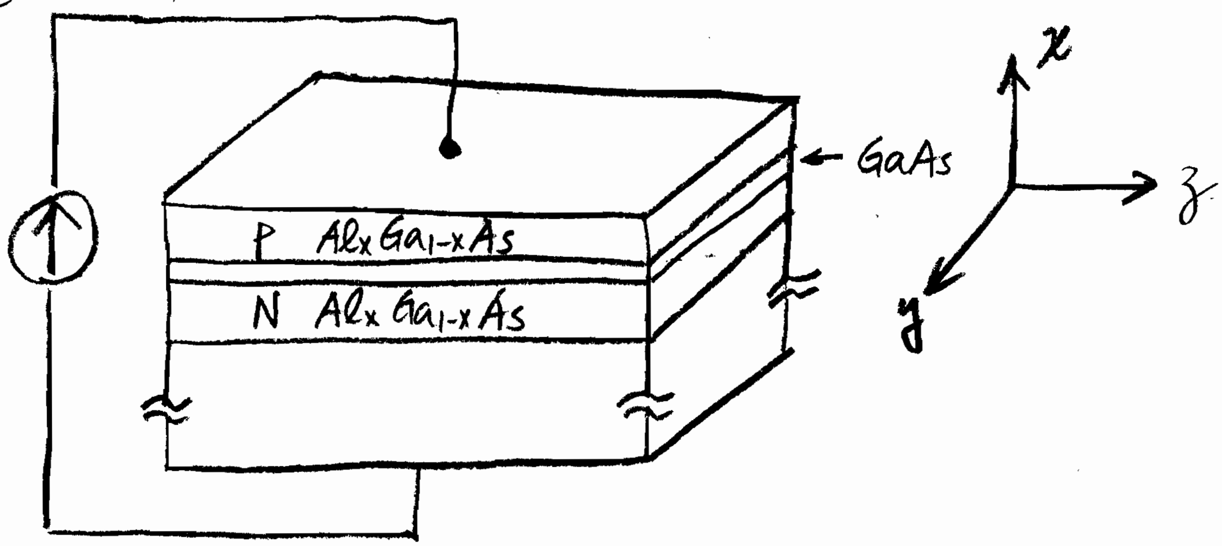
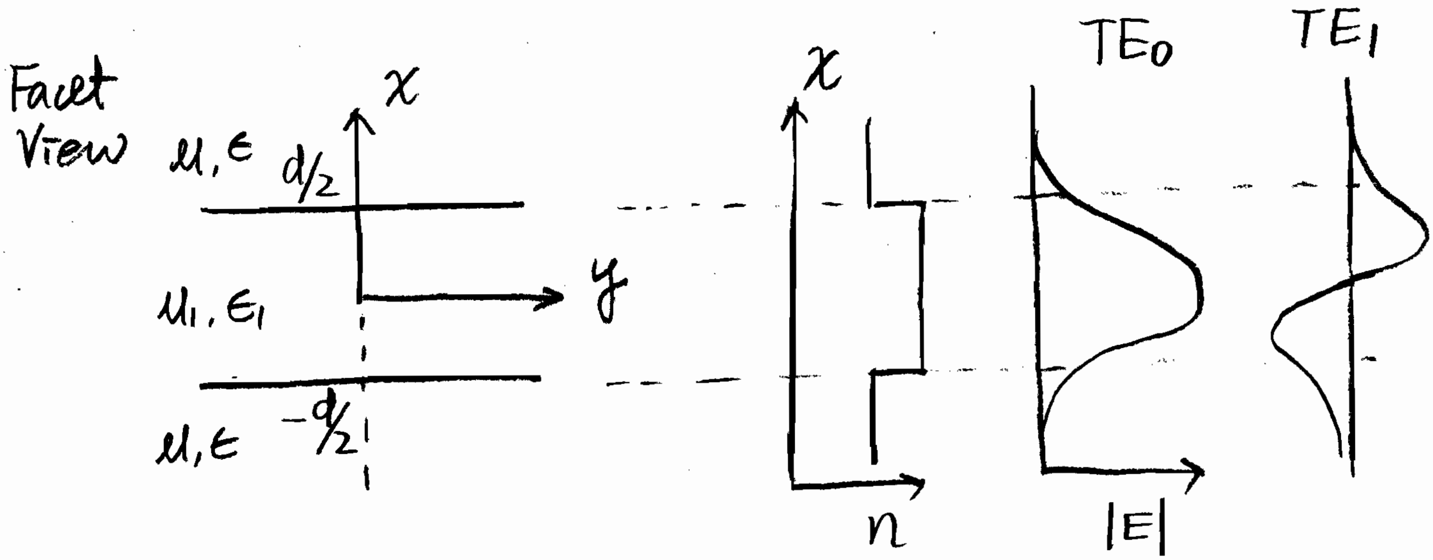
Optical Waveguide

DH forms a slab waveguide





TE (Transverse Electric) Modes

H_x, H_z $\quad \vec{E} = \hat{y} E_y \quad (E_x = E_z = 0)$

TM (Transverse Magnetic) Modes

E_x, E_z $\quad \vec{H} = \hat{y} H_y \quad (H_x = H_z = 0)$

Maxwell's Eq.

$$(\nabla^2 + \omega^2 \mu \epsilon)\vec{E} = 0$$

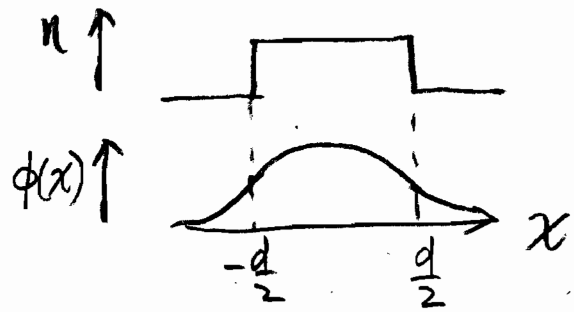
TE. $\vec{E} = \hat{y} E_y \quad (E_x = E_z = 0)$

$\frac{\partial}{\partial y} \to 0$ symmetry

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} + \omega^2 \mu \epsilon\right) E_y = 0$$

Similar to potential well solution

$$E_y = e^{ik_z z} \cdot \phi(x)$$



$$\phi(x) = \begin{cases} C_0 e^{-\alpha(|x| - \frac{d}{2})}, & |x| \geq \frac{d}{2} \\ C_1 \cos k_x \cdot x & |x| < \frac{d}{2} \end{cases}$$

↳ for even mode, such as TE_0, TE_2

→ $C_1 \sin k_x x$ for odd modes, TE_1, TE_3

$$\Rightarrow k_x^2 + k_z^2 = \omega^2 \mu_1 \epsilon_1 \quad \text{---} ①$$

$$-\alpha^2 + k_z^2 = \omega^2 \mu \epsilon \quad \text{---} ②$$

① − ②

$$k_x^2 + \alpha^2 = \omega^2 (\mu_1 \epsilon_1 - \mu \epsilon)$$

Matching boundary conditions

$$\begin{cases} E_x \text{ continuous} \\ H_z = \frac{1}{i\omega\mu} \cdot \frac{\partial E_y}{\partial x} \text{ continuous} \end{cases}$$

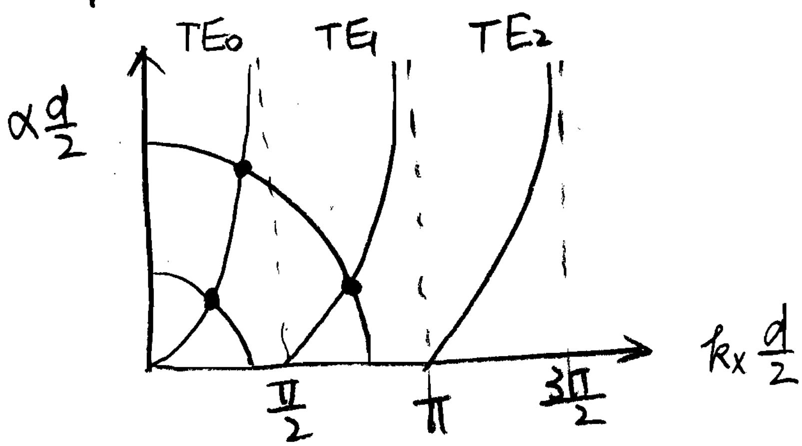
$$\Rightarrow \alpha = \frac{\mu}{\mu_1} k_x \tan\left(k_x \frac{d}{2}\right)$$

Compare with QW solution

$$\alpha = \frac{m_b}{m_w} k \tan\left(k \frac{L}{2}\right)$$

same eigenequation!

Graphic solution



① − ② : $k_x^2 + \alpha^2 = \omega^2 (\mu_1 \epsilon_1 - \mu \epsilon)$

$$= k_0^2 \cdot c^2 (\mu_1 \epsilon_1 - \mu \epsilon) \quad ; \quad c^2 = \frac{1}{\mu_0 \epsilon_0}$$

$$= k_0^2 \left(\frac{\mu_1 \epsilon_1}{\mu_0 \epsilon_0} - \frac{\mu \epsilon}{\mu_0 \epsilon_0} \right)$$

$$k_x^2 + \alpha^2 = k_0^2 (n_1^2 - n^2)$$

$$\left(k_x \frac{d}{2}\right)^2 + \left(\alpha \frac{d}{2}\right)^2 = \left(k_0 \frac{d}{2}\right)^2 (n_1^2 - n^2)$$

Single mode condition

$$\alpha \frac{d}{2} \rightarrow 0 , \quad k_x \frac{d}{2} = k_0 \frac{d}{2} \sqrt{n_1^2 - n^2} < \frac{\pi}{2}$$

$$n_1^2 - n^2 = (n_1+n)(n_1-n) \approx 2n_1 \Delta n$$

$$\Rightarrow \frac{2\pi}{\lambda_0}\sqrt{2n_1\Delta n} < \frac{\pi}{d}$$

$$\Delta n < \frac{1}{8n_1}\left(\frac{\lambda_0}{d}\right)^2 , \quad \text{or} \quad d < \frac{\lambda_0}{2\sqrt{2n_1\Delta n}}$$

Large d → small Δn

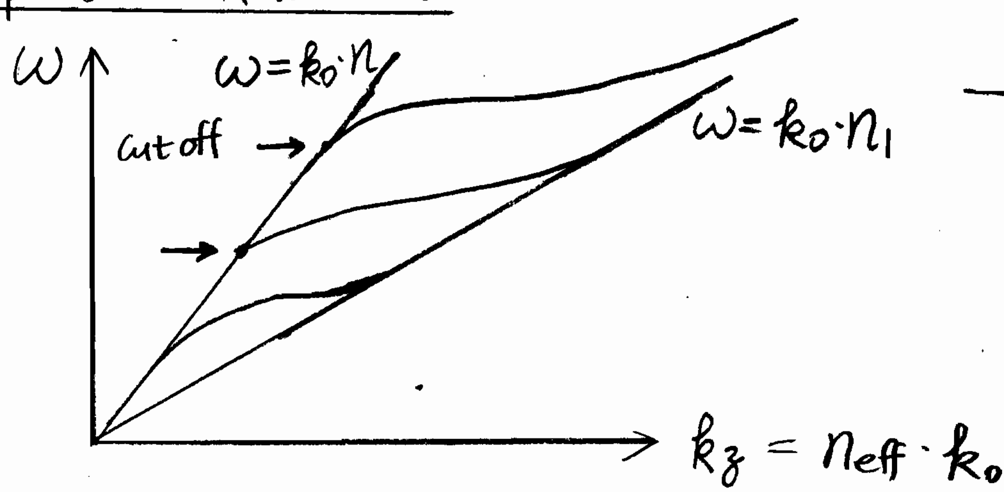
small d → Δn can be larger

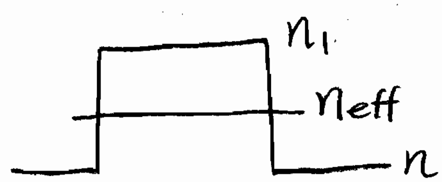
Example

$$\left\{\begin{array}{ll} \text{active layer GaAs.} & n_1 \sim 3.59 \\ \text{cladding " } Al_{0.3}Ga_{0.7}As, & n \sim 3.385 \end{array}\right\} \Delta n \sim 0.205$$

$$\Rightarrow d < \frac{0.87\ \mu m}{2\cdot\sqrt{2\times 3.59\times 0.205}} = 0.36\ \mu m$$

<u>Dispersion Relation.</u>





<Note this is ω-vs-k_z diagram. It's transpose of Fig. 7.4 in Chuang>

Optical Confinement Factor

$$\Gamma = \frac{\int_{core} \frac{1}{2} Re(\vec{E} \times \vec{H}^*)\, dx}{\int_{all} \frac{1}{2} Re(\vec{E} \times \vec{H}^*)\, dx}$$

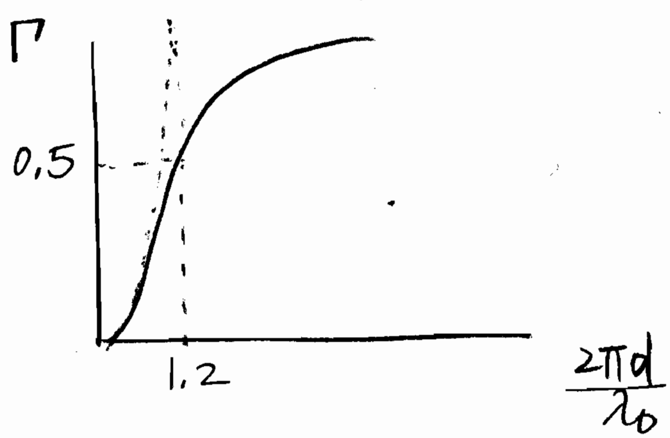
$$\vec{E} = \hat{y}\, E_y$$
$$\vec{H} = \hat{x}\, H_x = \hat{x} \cdot \left(-\frac{k_z}{\omega \mu}\right) E_y$$

If $\mu_1 = \mu$.

$$\Gamma = \frac{\int_{core} |\vec{E}|^2 dx}{\int_{-\infty}^{\infty} |\vec{E}|^2 dx}$$

For $\Gamma \ll 1$

$$\Gamma = \frac{1}{1 + \frac{2}{\alpha d}\left(\frac{\cos^2(k_x d/2)}{1 - \frac{\sin(k_x d)}{k_x d}}\right)} \longrightarrow 2\left(\frac{\pi d}{\lambda_0}\right)^2 (n_1^2 - n^2)$$



GaAs/AlGaAs

$$\begin{cases} n_1 = 3.59 \\ n = 3.385 \end{cases}$$

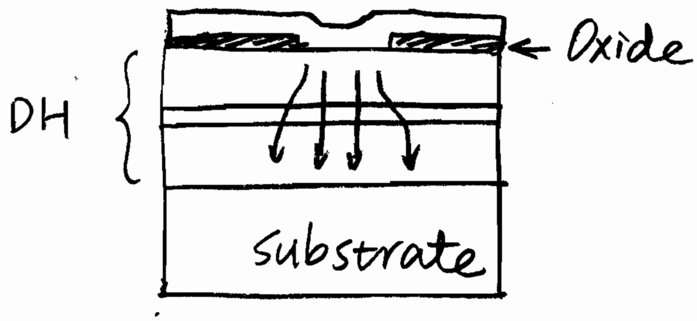
$\Gamma = 0.5$. $\frac{2\pi d}{\lambda_0} \approx 1.2$, $\lambda_0 \sim 0.87\,\mu m$, $d \approx 0.17\,\mu m$

(For $d = 0.17\,\mu m$. $\Gamma_{approx} \sim 1.08 \Rightarrow$ over-estimate)

Typical Laser Structures

(cross-sectional view at facet)

Gain-guided Laser



- Easy to fabricate
- Active region wider than oxide opening due to current spreading.

  $W_{eff} \sim 10\,um$

- $I_{th} \propto W$

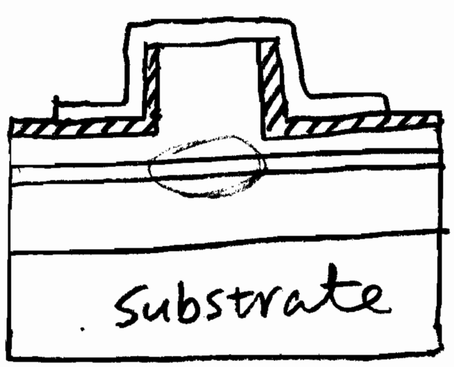
  → Higher threshold

- Mode

  → Easy to generate high-order lateral mode

Index-guided laser

① Ridge-waveguide laser



- Weak index guiding

  $\Delta n_{lateral} \sim 10^{-2}$

- Real index guiding

  → More stable mode

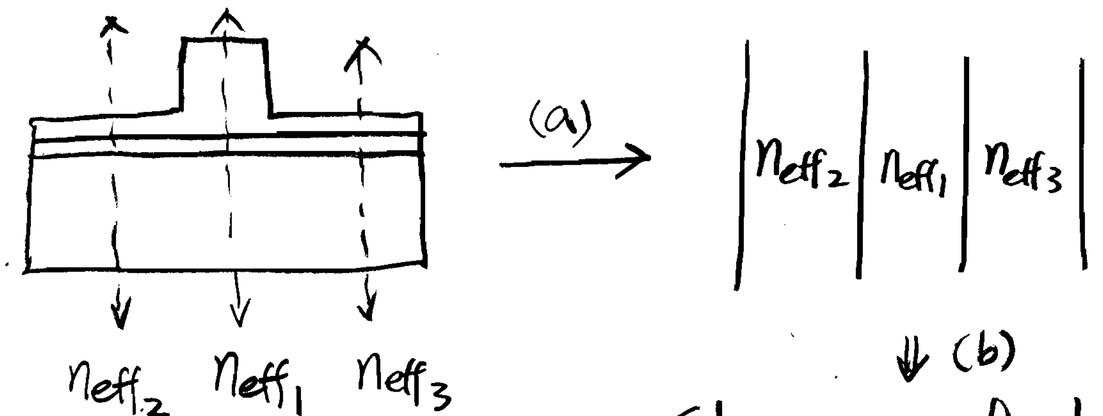
- Some current spreading
- Easy to fabricate

  → One epitaxial growth

Typical analysis for 2-D waveguide

Effective Index Method

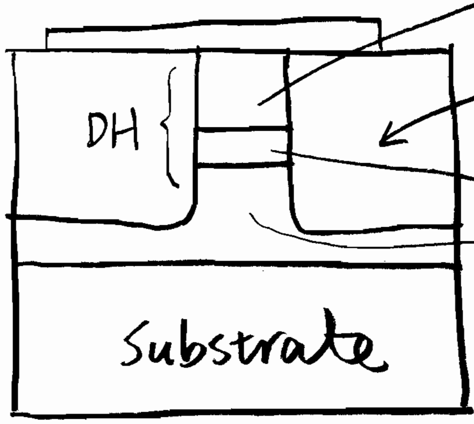
(a) Calculate n_{eff} along vertical cross-section at center and surrounding regions:



Solve vertical slab for final n_{eff}

(b) Solve vertical slab waveguide for n_{eff}

② Buried Heterostructure



- Heterostructures all around active region
- No current spreading → low I_{th}
- Tight lateral guiding (larger Δn)
- Narrow laser width, $w \sim 1\,\mu m$
  → low I_{th}
- Low parasitic capacitance → higher RC bandwidth
- Require multiple epi growth (2 or 3)
- Works well for InP/InGaAsP materials