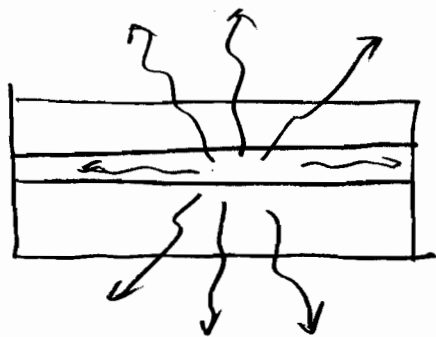


Amplified Spontaneous Emission (ASE)

LED



Spontaneous emission is amplified along the waveguide direction (still below threshold)

Spontaneous emission

Spectrum measured from top or bottom

$$L_w(\hbar\omega) = \hbar\omega \cdot \underbrace{r^{spont}(\hbar\omega)}_{\substack{\downarrow \\ \text{spontaneous emission} \\ \text{rate } [\frac{1}{\text{sec}} \cdot \frac{1}{\text{m}^3}]}} \cdot \underbrace{WdL}_{\substack{\downarrow \\ \text{active volume}}}$$

↑
window

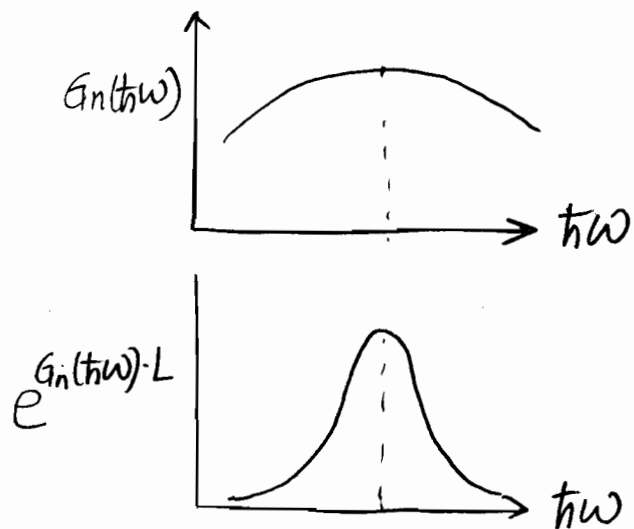
ASE

$$L_F(\hbar\omega) = \hbar\omega \int_{z=0}^L r^{spont}(\hbar\omega) e^{G_n(\hbar\omega) \cdot z} dz \cdot Wd$$

↑
Facet

$$G_n(\hbar\omega) = \Gamma g(\hbar\omega) - \alpha_c \quad : \text{Net modal gain}$$

$$L_F(\hbar\omega) = \hbar\omega \cdot r^{spont}(\hbar\omega) \cdot Wd \cdot \frac{e^{G_n(\hbar\omega)L} - 1}{G_n(\hbar\omega)}$$

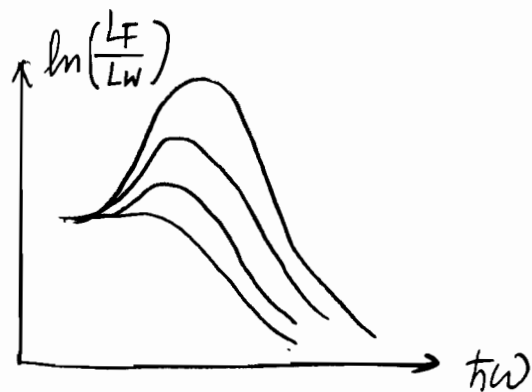
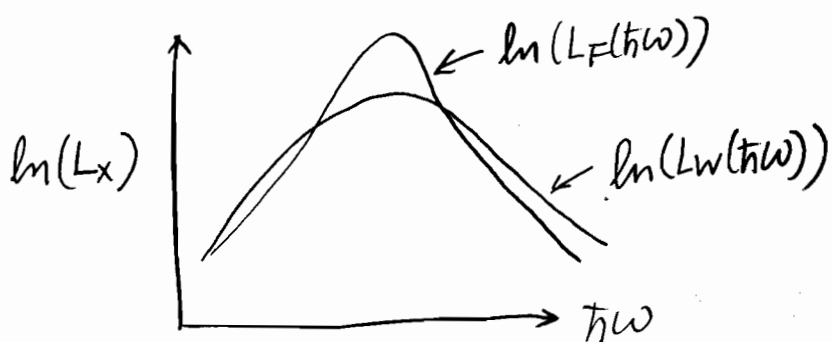


Spectrum measured from facets are narrower than that from top or bottom.

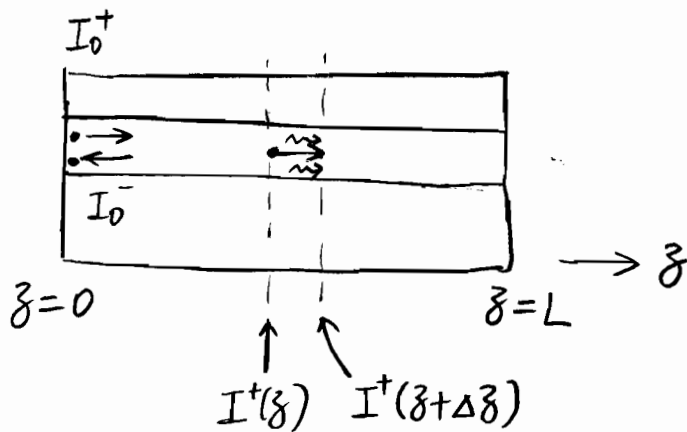
Measurement of Gain Spectrum:

$$\frac{L_F(h\nu)}{L_W(h\nu)} = \frac{e^{G_n(h\nu) \cdot L} - 1}{G_n(h\nu) \cdot L} \approx \frac{e^{G_n(h\nu) \cdot L}}{G_n(h\nu) \cdot L}$$

$$\ln\left(\frac{L_F(h\nu)}{L_W(h\nu)}\right) = G_n(h\nu) \cdot L - \ln(G_n(h\nu) \cdot L) \approx \overset{G_n L \gg 1}{G_n(h\nu) \cdot L}$$



Laser as Noise Amplifier



$$I^+(z+\Delta z) = I^+(z) \cdot e^{G_n \Delta z} + \frac{1}{2} \beta \cdot \gamma^{spont}(h\nu) \cdot h\nu \cdot \Delta z$$

$\frac{1}{2} \beta$ ↑
 spontaneous coupling factor:
 fraction of spontaneous coupled to
 lasing mode
 Forward
 or
 backward
 direction

For small Δz , $G_n \Delta z \ll 1$

$$I^+(z+\Delta z) = I^+(z) (1 + G_n \Delta z) + \frac{1}{2} \beta \cdot \gamma^{spont}(h\nu) \cdot h\nu \cdot \Delta z$$

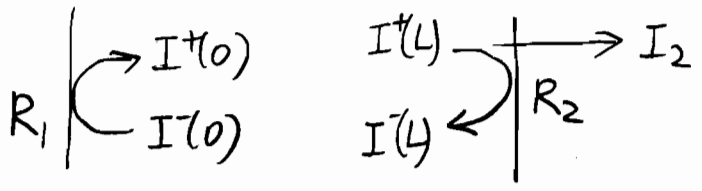
$$\Rightarrow \frac{dI^+(z)}{dz} = \frac{I^+(z+\Delta z) - I^+(z)}{\Delta z} = G_n I^+(z) + \underbrace{\frac{1}{2} \beta \gamma^{spont}(h\nu) \cdot h\nu}_{W(h\nu)}$$

Similarly for backward wave

$$\frac{dI^-(z)}{dz} = -G_n I^-(z) - W(h\nu)$$

Solutions:
$$\begin{cases} I^+(z) = I_0^+ e^{G_n z} - \frac{W(h\nu)}{G_n(h\nu)} \\ I^-(z) = I_0^- e^{-G_n z} - \frac{W(h\nu)}{G_n(h\nu)} \end{cases}$$

Boundary conditions:



$$\begin{cases} I^+(0) = R_1 I^-(0) \\ I^-(L) = R_2 I^+(L) \end{cases} \Rightarrow \text{solve } I_0^+ \text{ and } I_0^-$$

$$I_0^+ = \frac{(1 + R_1 e^{g_n L}) - R_1 (1 + R_2 e^{g_n L})}{1 - R_1 R_2 e^{2g_n L}} \cdot \frac{W(\hbar\omega)}{G(\hbar\omega)}$$

Output

$$\begin{aligned} I_2 &= (1 - R_2) \cdot I^+(L) \\ &= (1 - R_2) \cdot \frac{(e^{g_n L} - 1)(R_1 e^{g_n L} - 1)}{1 - R_1 R_2 e^{2g_n L}} \cdot \frac{W(\hbar\omega)}{G(\hbar\omega)} \end{aligned}$$

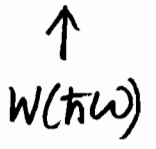
Threshold condition

$$\text{Denominator} = 0 \Rightarrow 1 - R_1 R_2 e^{2g_n L} = 0$$

Because of the presence of spontaneous emission $W(\hbar\omega)$, round-trip gain $R_1 R_2 e^{2g_n L} < 1$

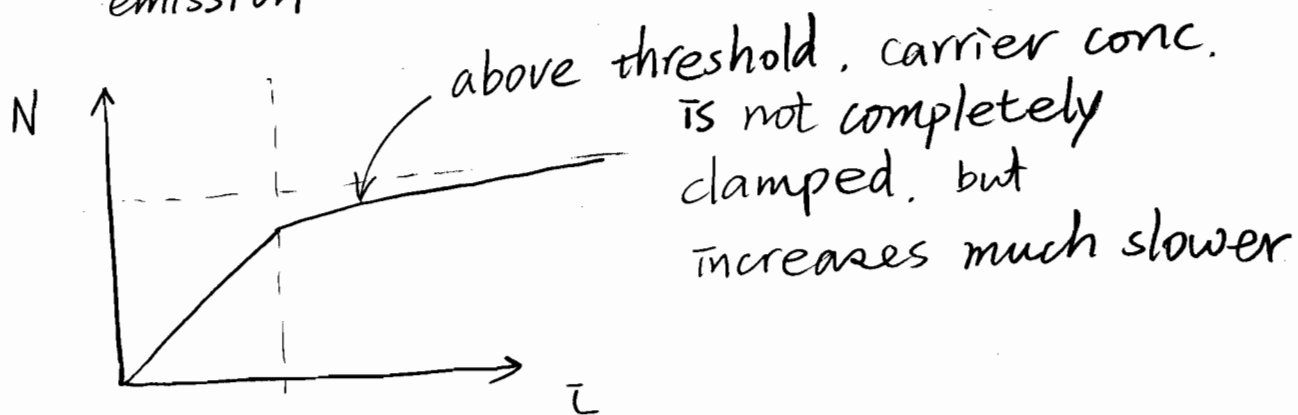
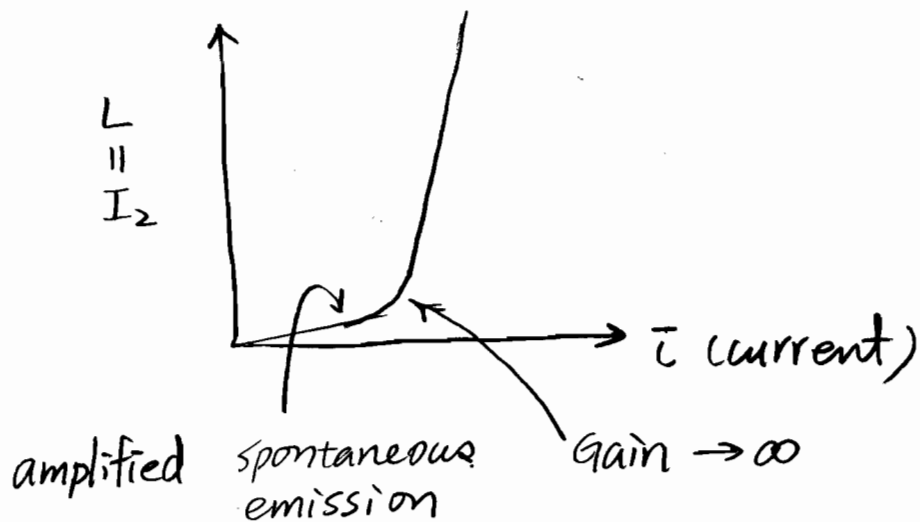
\Rightarrow Denominator > 0

Laser is a noise amplifier



$W(\hbar\omega)$

Refined model for L-I curve



Longitudinal Modes (§ 7.6)

81

$$\vec{E}(x, y, z) = \vec{E}_0(x, y) \cdot e^{i k_z z}$$

Round-trip phase condition

$$e^{i 2 k_z L} = 1$$

$$2 k_z L = 2\pi \cdot m \quad ; \quad m = \text{integer}$$

$$k_z = \frac{\omega}{c} \cdot n_1 \quad n_1 = \text{effective refractive index}$$

$$2 \cdot \frac{2\pi f_m}{c} \cdot n_1 L = 2\pi \cdot m$$

$$f_m = m \cdot \frac{c}{2 n_1 L} \quad \text{longitudinal modes}$$

Mode spacing

$$\Delta f = f_m - f_{m-1} = \frac{c}{2 n_1 L} + m \cdot \frac{c}{2L} \cdot \frac{(-1)}{n_1^2} \cdot \frac{\partial n_1}{\partial f} \Delta f$$

$$\Rightarrow \Delta f = \frac{c}{2 n_1 L} \cdot \left(1 + \underbrace{\frac{f}{n_1} \frac{\partial n_1}{\partial f}}_{\substack{\uparrow \\ \text{material dispersion}}} \right)^{-1}$$

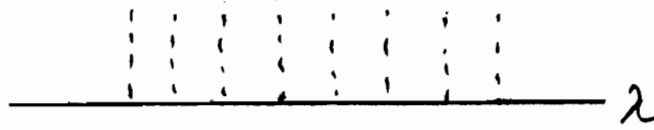
Or in terms of λ

$$m \cdot \lambda = 2 n_1 L$$

$$\Delta \left(m = \frac{2 n_1 L}{\lambda} \right) \Rightarrow 1 = \frac{-1}{\lambda^2} \Delta \lambda \cdot 2 n_1 L + \frac{2L}{\lambda} \cdot \frac{\partial n_1}{\partial \lambda} \Delta \lambda$$

$$\Delta \lambda = \frac{-\lambda^2}{2 n_1 L \left(1 - \frac{\lambda}{n_1} \frac{\partial n_1}{\partial \lambda} \right)}$$

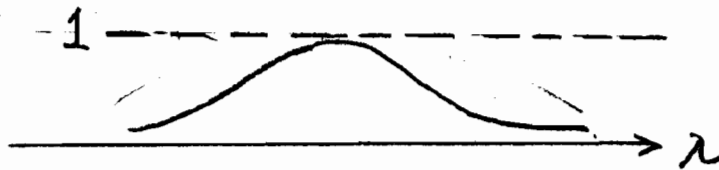
Modes



Gain

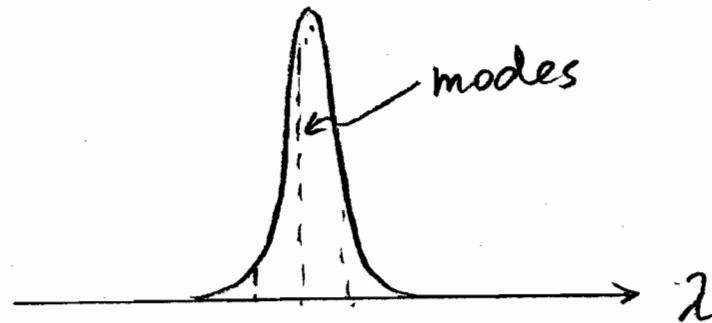


$R_1 R_2 e^{2G_n(\lambda)L}$

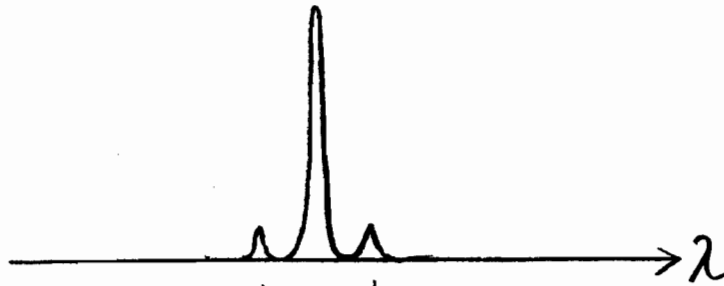


Amplification Factor

$$\frac{1}{1 - R_1 R_2 e^{2G_n \cdot L}}$$



Lasing Spectrum



Below Threshold

