Slowly Varying Envelop Approximation

$$H'_{ba} = -\vec{E}\cdot\vec{\mu}_{ba} = -\langle b|\vec{E}\cdot(-e\vec{r})|a\rangle$$

$$= -\frac{eE_0}{2}\hat{e}\cdot\int_V \psi_b^*(\vec{r})\, e^{i\vec{k}_{op}\cdot\vec{r}}\cdot\vec{r}\cdot\psi_a(\vec{r})\,d\vec{r}$$

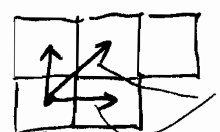
$$= -\frac{eE_0}{2}\hat{e}\cdot\int_V u_c^*(\vec{r})\, e^{-i\vec{k}_c\cdot\vec{r}}\cdot e^{i\vec{k}_{op}\cdot\vec{r}}\cdot\vec{r}\cdot u_v(\vec{r})\, e^{i\vec{k}_v\cdot\vec{r}}\,d\vec{r}\,\frac{1}{V}$$

$$= -\frac{eE_0}{2}\hat{e}\cdot\int_V u_c^*(\vec{r})\,\vec{r}\,u_v(\vec{r})\cdot e^{i\Delta\vec{k}\cdot\vec{r}}\,d\vec{r}\,\frac{1}{V}$$

$$\Delta\vec{k} = \vec{k}_{op} + \vec{k}_v - \vec{k}_c$$

$$= -\frac{eE_0}{2}\hat{e}\cdot\sum_n\int_\Omega u_c^*(\vec{r}+\vec{R}_n)\,(\vec{r}+\vec{R}_n)\cdot u_v(\vec{r}+\vec{R}_n)\cdot e^{i\Delta\vec{k}\cdot(\vec{r}+\vec{R}_n)}\,d\vec{r}\,\frac{1}{V}$$

* Divide integration to over unit cells



$\vec{R}_n$: lattice vector

unit cell, Ω

$$u_c(\vec{r}+\vec{R}_n) = u_c(\vec{r})$$
$$u_v(\vec{r}+\vec{R}_n) = u_v(\vec{r})$$

$e^{i\Delta\vec{k}\cdot\vec{r}}$ vary slowly within a unit cell.

$$\Rightarrow e^{i\Delta\vec{k}\cdot(\vec{r}+\vec{R}_n)} \approx e^{i\Delta\vec{k}\cdot\vec{R}_n}$$

$$= -\frac{eE_0}{2}\hat{e}\cdot\sum_n\int_\Omega u_c^*(\vec{r})\,(\vec{r}+\vec{R}_n)\cdot u_v(\vec{r})\,d\vec{r}\cdot e^{i\Delta\vec{k}\cdot\vec{R}_n}\,\frac{1}{V}$$

$$\int_\Omega u_c^*(\vec{r})\vec{R}_n\cdot u_v(\vec{r})\,d\vec{r} = \vec{R}_n\cdot\int_\Omega u_c^*(\vec{r})\,u_v(\vec{r})\,d\vec{r} = 0$$

$$= -\frac{eE_0}{2}\hat{e}\cdot\underbrace{\int_\Omega u_c^*(\vec{r})\cdot\vec{r}\,u_v(\vec{r})\,d\vec{r}\cdot\left[\frac{1}{\Omega}\right.}\ \underbrace{\left.\sum_n e^{i\Delta\vec{k}\cdot\vec{R}_n}\cdot\Omega\right]}\frac{1}{V}$$

$$= -\frac{eE_0}{2}\hat{e}\cdot\vec{r}_{cv}\cdot\int_V e^{i\Delta\vec{k}\cdot\vec{r}}\,\frac{d\vec{r}}{V}$$

$$= -\frac{eE_0}{2}\hat{e}\cdot\vec{r}_{cv}\cdot\delta_{\Delta\vec{k},0}$$