Absorption

Net absorption rate per unit volume within dE

$$R_{net}^{abs} = r_{net}^{abs}(E_{21})\,dE$$
$$= B_{12} f_1(1-f_2) P(E_{21}) - B_{21} f_2 (1-f_1) P(E_{21})$$
$$= B_{12}(f_1 - f_2) P(E_{21})$$

Absorption Coef:

$$\alpha(E_{21})\,dE = \frac{r_{net}^{abs}(E_{21})\,dE}{P(E_{21})\left(\frac{c}{n_r}\right)} = \frac{n_r}{c} \cdot B_{12}(f_1 - f_2)$$

Compare with earlier results.

$$\alpha = \frac{R}{I} = \frac{1}{I}\left(\frac{2}{V}\right) \cdot \sum_{k_a} \sum_{k_b} \frac{2\pi}{\hbar} |H'_{ba}|^2 \cdot \delta(E_b - E_a - \hbar\omega)(f_a - f_b)$$

$$I = \left(\frac{\epsilon_r \epsilon_0}{2} E_0^2\right)\left(\frac{1}{\hbar\omega}\right)\left(\frac{c}{n_r}\right) \quad \text{Photon flux}$$

For 2 discrete state

$$\alpha \to \frac{1}{I}\left(\frac{2}{V}\right) \cdot \frac{2\pi}{\hbar} |H'_{ba}|^2 \cdot (f_a - f_b)$$

$$\Rightarrow B_{12} = B_{21} = \frac{2 \cdot 2\hbar\omega}{V \cdot \epsilon_r \epsilon_0 E_0^2} \cdot |H'_{ba}|^2 = \left(\frac{2}{V}\right)\frac{2\hbar\omega}{\epsilon_r \epsilon_0 E_0^2} \cdot \frac{E_0^2}{4} \cdot |\hat{e} \cdot \vec{\mu}_{ab}|^2$$

$$= \frac{\hbar\omega}{V \cdot \epsilon_r \epsilon_0} |\hat{e} \cdot \vec{\mu}_{ab}|^2$$

Spontaneous Emission Spectrum

$$\frac{r_{21}^{spon}(E_{21})}{\alpha(E_{21})} = \frac{A_{21} f_2(1-f_1)}{\left(\frac{n_r}{c}\right) B_{12}(f_1-f_2)}$$

$$= \left(\frac{c}{n_r}\right) \cdot N(E_{21}) \cdot \frac{1}{e^{(E_{21}-\Delta F)/k_BT} - 1}$$

similar to derivation in Einstein's AB coef, but now with 2 separate quasi-Fermi levels. $\Delta F = F_2 - F_1$

$\uparrow E_{F2}$ $\uparrow E_{F1}$

$$r_{21}^{spon}(E_{21}) = \left(\frac{8\pi n_r^2 E_{21}^2}{h^3 c^2}\right)\left(\frac{1}{e^{(E_{21}-\Delta F)/k_BT} - 1}\right)\alpha(E_{21})$$

$\left[\frac{1}{s} \cdot \frac{1}{cm^3} \cdot \frac{1}{eV}\right]$

valid for nonequilibrium case

Stimulated Emission Spectrum

$$\frac{r_{net}^{stim}(E_{21})}{\alpha(E_{21})} = \frac{B_{21} \cdot P(E_{21})(f_2-f_1)}{\left(\frac{n_r}{c}\right) \cdot B_{12}(f_1-f_2)} = -\frac{c}{n_r} \cdot N(E_{21})$$

$n_{ph} = 1$ for monochromatic light

$$r_{net}^{stim}(E_{21}) = \frac{8\pi n_r^2 E_{21}^2}{h^3 c^2}\left[\underbrace{-\alpha(E_{21})}_{g(E_{21})\,:\,\text{gain}}\right]$$

$\left[\frac{1}{s} \cdot \frac{1}{cm^3} \cdot \frac{1}{eV}\right]$

Recap

$E = \hbar\omega.$

$$\alpha(\hbar\omega) = \frac{\pi\omega}{n_r c \epsilon_0} \frac{2}{V} \sum_{k_a} \sum_{k_b} |\hat{e}\cdot\vec{\mu}_{ab}|^2 \cdot \delta(E_b - E_a - \hbar\omega)\cdot(f_a - f_b)$$

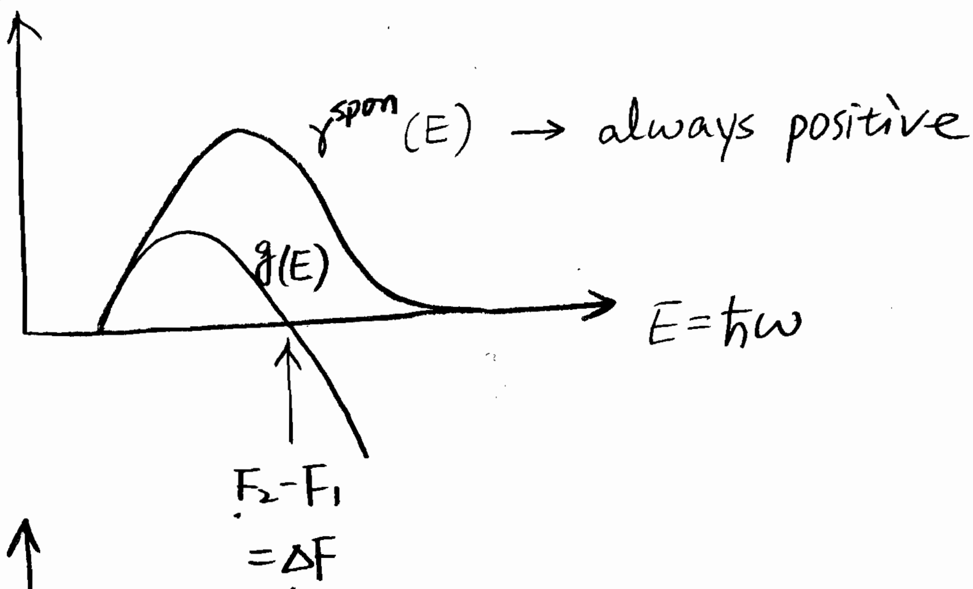
$$r^{spon}(\hbar\omega) = \frac{8\pi n_r^2 E^2}{h^3 c^2} \cdot \frac{1}{e^{(E-\Delta F)/k_B T} - 1} \cdot \alpha(\hbar\omega)$$

$$\gamma^{stim}_{net}(\hbar\omega) = \frac{8\pi n_r^2 E^2}{h^3 c^2} \cdot \left[-\alpha(\hbar\omega)\right]$$

$$\frac{\gamma^{stim}_{net}(\hbar\omega)}{r^{spon}(\hbar\omega)} = 1 - e^{(E-\Delta F)/k_B T}$$

$$g(E) = -\alpha(E)$$

Spontaneous Emission & Gain spectra



$$\frac{-1}{e^{(E-\Delta F)/k_B T} - 1}$$