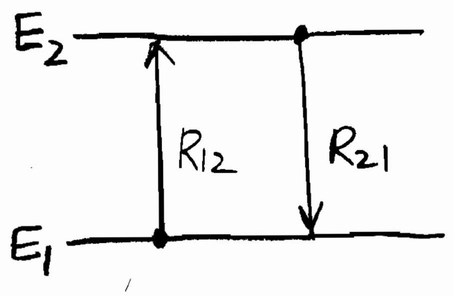
Einstein's AB coefficients (1916)



Upward transition (Absorption)

$$R_{12} = B_{12} P(E_{21}) \cdot f_1 (1-f_2) \qquad \left[\text{unit: } \frac{1}{cm^3 \cdot sec}\right]$$

$$P(E_{21}) = N(E_{21})\, n_{ph}$$

↑ photon density of states

↑ Photons/state

$$\begin{cases} N(E_{21}) = \dfrac{8\pi n_r^3 E_{21}^2}{h^3 c^3} \\[2ex] n_{ph} = \dfrac{1}{e^{E_{21}/k_B T} - 1} \end{cases}$$

Downward transition (Emission)

* Spontaneous emission

$$R_{21}^{spon} = A_{21} \cdot f_2 (1-f_1)$$

* Stimulated emission

$$R_{21}^{stim} = B_{21} P(E_{21}) \cdot f_2 (1-f_1)$$

At thermal equilibrium

$$R_{12} = R_{21}$$

$$R_{12} = R_{21}^{stim} + R_{21}^{spon}$$

$$B_{12} f_1 (1-f_2) \cdot \underline{\underline{P(E_{21})}} = B_{21} f_2 (1-f_1)\, \underline{\underline{P(E_{21})}} + A_{21} f_2 (1-f_1)$$

$$P(E_{21}) = \frac{A_{21}\, f_2(1-f_1)}{B_{12} f_1(1-f_2) - B_{21} f_2(1-f_1)}$$

$$f_1 = \frac{1}{e^{(E_1-E_F)/k_BT}+1} \qquad f_2 = \frac{1}{e^{(E_2-E_F)/k_BT}+1}$$

Products of all denominators

$$DE = [e^{(E_1-E_F)/k_BT}+1][e^{(E_2-E_F)/k_BT}+1]$$

$$f_2(1-f_1) = \frac{1 \cdot e^{(E_1-E_F)/k_BT}}{DE}$$

$$f_1(1-f_2) = \frac{1 \cdot e^{(E_2-E_F)/k_BT}}{DE}$$

$$P(E_{21}) = \frac{A_{21}}{B_{12}\left[\frac{f_1(1-f_2)}{f_2(1-f_1)}\right] - B_{21}} = \frac{A_{21}}{B_{12}\, e^{E_{21}/k_BT} - B_{21}}$$

$$= N(E_{21}) \cdot \frac{1}{e^{E_{21}/k_BT}-1}$$

$$\Rightarrow \boxed{\begin{array}{l} B_{12} = B_{21} \\ \dfrac{A_{21}}{B_{21}} = N(E_{21}) = \dfrac{8\pi n_r^3 E_{21}^2}{h^3 c^3} \end{array}}$$

$$\frac{R_{21}^{stim}}{R_{21}^{spon}} = \frac{B_{21}\,P(E_{21})}{A_{21}} = \frac{P(E_{21})}{N(E_{21})} = \frac{N(E_{21})\cdot n_{ph}}{N(E_{21})} = n_{ph}$$

At equlibrium, the ration of stimulated and spon emission is the number of photons with energy $= E_{21}$