Electron-Photon Interaction

In the absence of photon field

$$H_0 = \frac{1}{2m_0} p^2 + V(\vec{r})$$

In the presence of photon (electromagnetic) field

$$\vec{P} \to (\vec{P} - e\vec{A}) \qquad \vec{A} : \text{vector potential}$$

$$\nabla \times \vec{A} = \vec{B}$$

$$\vec{E} = -\frac{\partial \vec{A}}{\partial t}$$

$$H = \frac{1}{2m_0}(\vec{P} - e\vec{A})^2 + V(\vec{r})$$

$$= \frac{1}{2m_0} p^2 - \frac{e}{2m_0}(\vec{P}\cdot\vec{A} + \vec{A}\cdot\vec{P}) + \frac{e^2A^2}{2m_0} + V(\vec{r})$$

$$= H_0 + H'$$

$$H' = -\frac{e}{2m_0}(\vec{p}\cdot\vec{A} + \vec{A}\cdot\vec{P}) + \cancel{\frac{e^2A^2}{2m}}$$

$$|eA| \ll |P|$$

$$\vec{p}\cdot\vec{A}\,\phi = -i\hbar\nabla\cdot\vec{A}\phi = -i\hbar[(\nabla\cdot\vec{A})\phi + \underbrace{\vec{A}\cdot\nabla\phi}_{\downarrow\ \vec{A}\cdot\vec{P}}]$$

Use Coulumb gauge $\nabla\cdot\vec{A} = 0$

$\vec{p}\cdot\vec{A} = \vec{A}\cdot\vec{P}$ operator

$$H' \approx -\frac{e}{m_0}\vec{A}\cdot\vec{P}$$

$$= H'(\vec{r})e^{-i\omega t} + H'^{+}(\vec{r})e^{i\omega t}$$

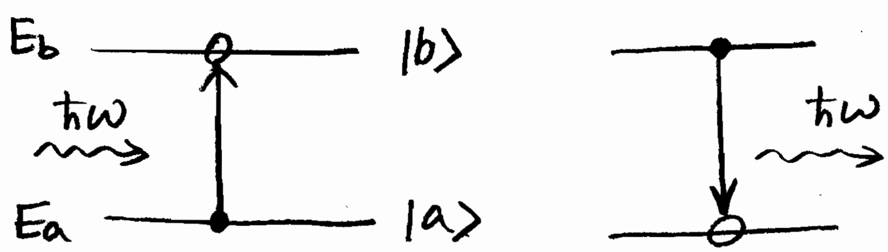
Assume

$$\vec{A} = \hat{e}\, A_0 \cos(\vec{k}_{op}\cdot\vec{r} - \omega t)$$
$$= \hat{e}\,\frac{A_0}{2}\, e^{i\vec{k}_{op}\cdot\vec{r} - i\omega t} + \hat{e}\,\frac{A_0}{2}\cdot e^{-i\vec{k}_{op}\cdot\vec{r} + i\omega t}$$

$$\Rightarrow\ H'(\vec{r}) = -\frac{eA_0\, e^{i\vec{k}_{op}\cdot\vec{r}}}{2m_0}\,\hat{e}\cdot\vec{p}$$

Absorption Rate of one photon

$$W_{abs} = \frac{2\pi}{\hbar}\,|\langle b|H'(\vec{r})|a\rangle|^2\cdot\delta(E_b - E_a - \hbar\omega)$$



Absorption　　　　Emission

$$W_{emi} = \frac{2\pi}{\hbar}\,|\langle a|H'(\vec{r})|b\rangle|^2\,\delta(E_a - E_b + \hbar\omega)$$

Total rate of upward transition per unit vol.

$$R_{a\to b} = \frac{2}{V}\sum_{k_a}\sum_{k_b}\frac{2\pi}{\hbar}\,|H'_{ba}|^2\cdot\delta(E_b - E_a - \hbar\omega)\cdot f_a\cdot(1-f_b)$$

$f_a = \dfrac{1}{1+e^{(E_a-E_F)/k_BT}}$: Probability $|a\rangle$ occupied

$(1-f_b)$: " $|b\rangle$ empty

Total downward transition

$$R_{b\to a} = \frac{2}{V}\sum_{k_a}\sum_{k_b}\frac{2\pi}{\hbar}|H'_{ab}|^2\,\delta(E_a - E_b + \hbar\omega)\cdot f_b(1-f_a)$$

$$|H'_{ab}| = |H'_{ba}|$$

"Net" upward transition rate (absorption).

$$R = R_{a\to b} - R_{b\to a}$$

$$= \frac{2}{V}\sum_{k_a}\sum_{k_b}\frac{2\pi}{\hbar}|H'_{ba}|^2\,\delta(E_b - E_a - \hbar\omega)\cdot(f_a - f_b)$$

$$* \; f_a(1-f_b) - f_b(1-f_a) = f_a - f_b$$

$\left(\text{unit: } \frac{1}{cm^3}\cdot\frac{1}{sec}\right)$

Matrix Element H'_{ba}

$$H'_{ba} = \langle b|\left(-\frac{e}{m_0}\vec{A}\cdot\vec{P}\right)|a\rangle$$

$$= \frac{-e}{m_0}\vec{A}\cdot\langle b|\vec{P}|a\rangle$$

$$\vec{P} = m_0\frac{d}{dt}\vec{r} = m_0\frac{1}{i\hbar}[\vec{r}, H_0]$$ Heisenberg Picture

$$= \frac{m_0}{i\hbar}(\vec{r}H_0 - H_0\vec{r})$$

$$\langle b|\vec{P}|a\rangle = \frac{m_0}{i\hbar}\left(\langle b|\vec{r}\underbrace{H_0|a\rangle}_{E_a|a\rangle} - \underbrace{\langle b|H_0}_{E_b\langle b|}\vec{r}|a\rangle\right)$$

$$= \frac{m_0}{i\hbar}\underbrace{(E_a - E_b)}_{-\hbar\omega}\cdot\langle b|\vec{r}|a\rangle$$

$$H'_{ba} = \left(-\frac{e}{m_0}\right)\left(-\frac{m_0}{i\hbar}\right)\hbar\omega\,\vec{A}\cdot\vec{r}_{ba}$$

$$= (-i\omega\vec{A})\cdot\underbrace{(e\vec{r}_{ba})}_{\text{electric dipole}} \qquad \vec{E} = -\frac{\partial}{\partial t}\vec{A} = i\omega\vec{A}$$

$$= -\vec{E}\cdot\vec{\mu}_{ba}\,, \qquad \vec{E} = \hat{e}\left(\frac{E_0}{2}e^{i\vec{k}_{op}\cdot\vec{r}-i\omega t} + \frac{E_0}{2}e^{-i\vec{k}_{op}\cdot\vec{r}+i\omega t}\right)$$

$$R = \frac{2}{V}\sum_{k_a}\sum_{k_b}\frac{2\pi}{\hbar}\frac{E_0^2}{4}\left|\hat{e}\cdot\vec{\mu}_{ba}\right|^2\cdot\delta(E_b - E_a - \hbar\omega)\,(f_a - f_b)$$

$\uparrow$ $\frac{1}{cm^3}\cdot\frac{1}{sec}$

Absorption Coefficient α (cm^{-1})

$$I = I_0\, e^{-\alpha z}$$

$\uparrow$ Photon flux $\frac{1}{cm^2}\cdot\frac{1}{sec}$

$$\left|\frac{dI}{dz}\right| = R = \alpha\cdot I$$

$$\alpha = \frac{R}{I}$$

$$I = \left(\frac{\epsilon_r\epsilon_0}{2}E_0^2\right)\left(\frac{1}{\hbar\omega}\right)\left(\frac{c}{n_r}\right)$$

$$\alpha = \frac{2\pi \cancel{E_0^2}}{\cancel{4}\hbar}\cdot\frac{\cancel{2}\cancel{\hbar}\omega\cdot n_r}{\epsilon_r\epsilon_0\cancel{E_0^2}\cdot c}\,\frac{2}{V}\sum_{k_a}\sum_{k_b}\left|\hat{e}\cdot\vec{\mu}_{ba}\right|^2\cdot\delta(E_b - E_a - \hbar\omega)(f_a - f_b)$$

$$\epsilon_r = n_r^2$$

$$\alpha = \frac{\pi\omega}{n_r c\,\epsilon_0}\,\frac{2}{V}\sum_{k_a}\sum_{k_b}\left|\hat{e}\cdot\vec{\mu}_{ab}\right|^2\cdot\delta(E_b - E_a - \hbar\omega)\cdot(f_a - f_b)$$