HW #7
Due November 30 (Wed) in the IN Box at 261M Cory

1. A distributed Bragg reflector (DBR) with a grating coupling coefficient of $\kappa$ and length of $L = 100 \, \mu m$. Assume the end facet of the grating is perfectly anti-reflection (AR)-coated. The grating reflectivity is defined as the field reflection coefficient at the interface of the grating and the input waveguide. Assume the average refractive indices are the same for the waveguide and the grating section so there is no reflection due to index discontinuity. All the reflection comes from the DBR. The Bragg wavelength is 1550 nm, and the effective refractive index of the waveguides is 3.5.

   a. Find the relation between $\Lambda = \frac{2m\pi}{\lambda} - \frac{\pi}{\delta n^2}$ and the deviation of wavelength from the Bragg wavelength $\Delta\lambda = \lambda - \lambda_{Bragg} = \lambda - 2n\Lambda$.

   b. Plot the magnitude (absolute value) and phase of the field reflectivity of the DBR versus $\Delta\lambda$ for the following three values of $\kappa$. The unit for $\Delta\lambda$ should be nm, and the range should be from -15 nm to +15 nm.
      i. $\kappa = 50 \, \text{cm}^{-1}$
      ii. $\kappa = 200 \, \text{cm}^{-1}$
      iii. $\kappa = 400 \, \text{cm}^{-1}$

   c. For each curve in Part b), calculate and mark the wavelength deviation $\Delta\lambda_\kappa$ that corresponds to $\delta = \pm \kappa$. As we discussed in lecture, the width of the reflection band is approximately $\kappa \rightarrow \kappa = 2\kappa$ for strong gratings (i.e., large $\kappa$). For the three $\kappa$ values, which one(s) can be considered strong grating? What are the values of $\kappa L$ product? (Hint: use the relation you find in Part a).

2. A quantum well vertical cavity surface-emitting laser (VCSEL) has the following parameters:
   - Optical gain: $g(N) = g_0 \ln(N/N_\tau)$, where $g_0 = 2000 \, \text{cm}^{-1}$, $N_\tau = 10^{18} \, \text{cm}^{-3}$
   - Intrinsic loss $\alpha_i = 5 \, \text{cm}^{-1}$
   - Laser area: 1x1 $\mu m^2$
   - Effective cavity length = 1 $\mu m$
   - Active region with 3 Quantum wells, each with thickness = 10 nm, located in the middle of the cavity at the peak of the optical standing wave
   - Facet reflectivity = 99.95% for the bottom DBR, and 99.5% for the top DBR
   - Internal quantum efficiency $\eta_i = 100\%$
   - Assume the total carrier lifetime at threshold = 1 nsec
   - Laser wavelength = 1240 nm

   a. Find the threshold gain of the VCSEL.
   b. Find the threshold current and external quantum efficiency from the top emission only.
   c. Construct the L-I curve for up to 3 times threshold current.
3. Consider a DH edge-emitting laser with the following parameters:

Optical gain: 
\[
g(N, S) = \frac{g'(N - N_r)}{1 + \varepsilon S}
\]
where 
\[
g' = 2 \times 10^{-16} \text{ cm}^2, \quad N_r = 10^{18} \text{ cm}^{-3}, \quad \text{and} \quad \varepsilon = 10^{-16} \text{ cm}^3
\]

Intrinsic loss \( \alpha_i = 10 \text{ cm}^{-1} \)

Laser width = 1 \( \mu \)m
Laser cavity length = 200 \( \mu \)m
Thickness of active layer = 0.1 \( \mu \)m
Confinement factor of active region = 50%
Facet reflectivity = 30% for both facets
Internal quantum efficiency \( \eta_i = 100\% \)

Assume the total carrier lifetime at threshold = 1 nsec
Laser wavelength = 1240 nm
Effective refractive index = 3

a. Find the threshold gain and photon lifetime
b. Find the photon density at 1 mW output power (from both facets)
c. Find the relaxation oscillation frequency at 1 mW output power
d. Find the K factor of the laser (express your answer in nsec).
e. Express the damping in terms of the K factor, plot the small-signal frequency response of the laser, i.e., 
\[
10 \cdot \log(|H(\omega)|^2)
\]
, at three output levels: (i) 1 mW, (ii) 10 mW, (iii) 100 mW.