HW #5
Due November 7 (Monday) in class

You need to use numeric packages (Matlab, MathCAD, Mathmatica, etc) for this HW.

1. The homework problem is an extension of the class lecture on the DC characteristics of semiconductor lasers for both below and above threshold. When we derive the light-versus-current (L-I) characteristics, we ignore the spontaneous emission term. Here, we show that the laser output is basically amplified spontaneous emission, and that when we consider the spontaneous emission term, the carrier concentration is very close to, but never reaches, the threshold carrier concentration derived in class. The L-I curve at threshold increase smoothly rather than abruptly.
The rate equations we discussed in class are:
$$\frac{dN}{dt} = \eta_i \frac{I}{qV_{active}} - \frac{N}{\tau} - \frac{c}{n_r} g(N) \cdot S$$
$$\frac{dS}{dt} = \frac{c}{n_r} \Gamma \cdot g(N) \cdot S - \frac{S}{\tau_p} + \beta \cdot R_{sp}$$
where
N is carrier concentration,
S is the photon density, both in $1/m^3$;
V_{active} is the volume of the active layer (i.e., laser width x length x thickness);
τ is the total carrier lifetime, $\frac{1}{\tau} = \frac{1}{\tau_r} + \frac{1}{\tau_{nr}}$, τ_r is radiative recombination lifetime, and τ_{nr} is nonradiative recombination lifetime;
$\frac{c}{n_g}$ is the speed of light in semiconductor;
$g(N) = a \cdot (N - N_{tr})$ is the gain coefficient. Here, we use linear gain approximation.
Γ is the optical confinement factor;
$\tau_p = \frac{1}{\frac{c}{n_r}(\alpha_m + \alpha_i)}$ is the photon lifetime;
is the spontaneous emission factor, i.e., the fraction of the spontaneous emission coupled to the waveguide mode; and
$R_{sp} = \frac{N}{\tau_r} = \eta_r \frac{N}{\tau}$ is the spontaneous rate.

   a. In steady state, show that $S(N) = \dfrac{\beta \cdot R_{sp}(N)}{\frac{1}{\tau_p} - \frac{c}{n_r}\Gamma \cdot g(N)}$ for both below threshold and above threshold.

b. In steady state, show that $I(N)=\frac{qV_{active}}{\eta_i}\left(\frac{N}{\tau}+\frac{c}{n_r}g(N)\cdot S(N)\right)$ for both below and above threshold.

c. Show that the output power $P(N)=\hbar\omega\cdot\frac{V_{active}}{\Gamma}\alpha_m\frac{c}{n_r}\cdot S(N)$.

d. Since both output power *P(N)* and current *I(N)* are both functions of *N*, we can plot the light-versus-current (L-I) curve using *N* as a parameter. Use the following parameters for the plot:

$R_1=R_2=30\%$

$N_{tr}=10^{24}\ 1/\mathrm{m}^3$

$a=10^{-20}\ \mathrm{m}^2$

$\Gamma=50\%$

$\alpha_i=1000\ \mathrm{m}^{-1}$

$L=300\ \mu\mathrm{m}$

$w=1\ \mu\mathrm{m}$

$\mathrm{t}=1\ \mu\mathrm{m}$

$\tau=1\ \mathrm{nsec}$

$\eta_\mathrm{i}=100\%$

$\eta_r=90\%$

$\beta=10^{-2}$

Plot both L-I curve (i.e., *P(N)* versus *I(N)*) for output power from 0 to ~ 10 mW (i.e., choose the *N* values so that the output power goes from 0 to 10 mW). **Please note that the $N < N_{th}$ even above threshold, though it is getting closer and closer to N_{th} above threshold. To show clearly the behavior around threshold, you need to use a very fine interval for *N*, or alternatively, you can use non-uniform intervals with much denser points near N_{th}.**

e. Plot the carrier concentration *N* as a function of current *I* for the same range of *I* as in part d.

f. Plot *L-I* and *N-I* again using log scale for the vertical axes.

2. To show effect of β on the *L-I* characteristics, plot *L-I* for $\beta = 10^{-2}$, 10^{-3}, and 10^{-4} in the same graph. Choose the ranges of *N* such that the output power reaches 10 mW. Likewise, plot *N-I* curve for the three β values. Show your plots in log-linear and linear-linear plots for both families of curves.

3. For the laser in Problem 1, if its peak wavelength is 1550 nm,

   a. Find the mode spacing of the laser. Ignore the material dispersion for this problem (i.e., $dn_r/d\lambda=0$).

   b. Find the wavelengths of the modes immediately around 1550 nm.

4. To illustrate the spectral distribution of Fabry-Perot lasers, calculate the power of the three main modes described in Problem 3 (i.e., 1550 nm and the lower and upper modes). Follow the procedures outlined below:
   - Modify the gain coefficient to be wavelength dependent:

     $$g(N,\lambda) = a \cdot (N - N_{tr})\left(1 - \left(\frac{\lambda - \lambda_p}{\Delta\lambda}\right)^2\right)$$

     where $\lambda_{\mathrm{p}} = 1550\,\mathrm{nm}, \Delta\lambda = 200\,\mathrm{nm}$
   - The photon density for the three modes are

     $$S_0(N) = \frac{\beta \cdot R_{sp}(N)}{\frac{1}{\tau_p} - \frac{c}{n_r}\Gamma \cdot g(N,\lambda_p)}$$

     $$S_{-1}(N) = \frac{\beta \cdot R_{sp}(N)}{\frac{1}{\tau_p} - \frac{c}{n_r}\Gamma \cdot g(N,\lambda_{p-1})}$$

     $$S_{+1}(N) = \frac{\beta \cdot R_{sp}(N)}{\frac{1}{\tau_p} - \frac{c}{n_r}\Gamma \cdot g(N,\lambda_{p+1})}$$

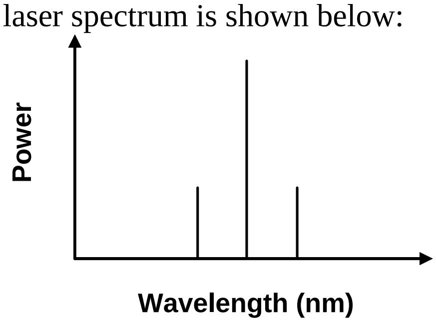
   where λ_{p-1} and λ_{p+1} are the lower and higher adjacent modes in wavelength.

   a. Show that the current expression is now modified to

   $$I(N) = \frac{qV_{active}}{\eta_i}\left(\frac{N}{\tau} + \frac{c}{n_r} g(N) \cdot \left(S_0(N) + S_{-1}(N) + S_{+1}(N)\right)\right)$$

   b. Plot the spectra of the laser for three cases:
      i. $I = 0.5 I_{th}$
      ii. $I = 1.0 I_{th}$
      iii. $I = 2 \cdot I_{th}$

   The qualitative feature of a laser spectrum is shown below:



   c. Plot the spectra in b. in log scale (for Y axes).