This semester's final exam will allow the candidate to bring with him one two-sided sheet of notes, but no other notes nor texts. Here is a list of topics and class notes about which exam questions may be posed.

## Vector spaces:

Abstract linear spaces, dimension, basis
Dual spaces, inner/scalar product, outer product/dyad
Subspaces, sums and intersections, spans and orthogonal complements.
Cross-product in Euclidean 3-space

## Abstract Linear Maps/Transformations:

Domain, codomain/target-space, kernel/nullspace, range
Sums and products of linear maps, inverses
Representation by matrices dependent upon bases
Change of basis, canonical bases ( anticipating later developments )

## Elementary row and column reductions to canonical forms

Row echelon form, column echelon form, diagonal canonical form under Equivalence
Rank, equality of row rank and column rank, nullity
Triangular factorizations and variants of Gaussian Elimination, Fredholm's Alternatives

## Determinants

Determinant as ratio of volumes, obtainable from triangular factors
Determinantal expansions and rank, Cramer's rule, Adjugate, Jacobi's formula for derivative

## Convexity

Convex body as convex hull of points, as intersection of half-spaces
Support planes, separating planes

## Normed linear spaces

Vector norms, triangle inequality, convergence, completeness, compactness
Dual norms, operator/matrix norms, projections
Nearness to singularity, norm of inverse, ill conditioned linear systems
Euclidean and Unitary spaces, orthogonal maps, transpose of matrix
Gram-Schmidt orthogonalization, positive definite matrices, Cholesky factorization
Least Squares, Linearly constrained least squares

## Eigenvalues and Eigenvectors

Triangularization by similarity, block triangularization
Characteristic polynomial, Cayley-Hamilton theorem
Jordan's normal form, irreducible invariant subspaces, continuity and derivatives of eigenvalues
Real symmetric matrices, variational derivation of eigenvalues, interlacing properties
Singular value decomposition

An important goal of the course is the capability to read and figure out mathematical material about linear algebra and matrices even if it has not been seen before.

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## List of Relevant Class Notes

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|---|---|
| Cross.pdf | Cross-Products and Rotations in Euclidean 2- and 3-Space. |
| Axioms.pdf | Axioms for Fields and Vector Spaces. |
| prblms1.pdf | Solutions to Problems about Axioms for Fields and Vector Spaces. |
| 2dspaces.pdf | Notes on 2-Dimensional Spaces. |
| RREF1.pdf | The Reduced Row-Echelon Form is Unique. |
| pts.pdf | Vector Spaces, Bases, and Dual Spaces. |
| geo.pdf | Geometry of Elementary Operations. |
| geos.pdf | Geometry of Elementary Operations and Subspaces. |
| s10oct.pdf | Solutions to problems issued 10 Oct. 2000. |
| lstsqrs.pdf | Least-Squares and Bilinear Forms. |
| prblms2.pdf | Solutions to Problem Set 2 (Fall 1998). |
| tkhms.pdf | Solutions for Take-Home Test due 5 Oct. 1998. |
| jacobi.pdf | Jacobi's formula for the derivative of a determinant. |
| gji.pdf | Gauss-Jordan Inversion of Matrices. |
| chio.pdf | Chio's Trick for Linear Equations with Integer Coefficients. |
| gcd5.pdf | Euclid's GCD Algorithm for Linear Equations with Integer Coefficients. |
| diagprom.pdf | Diagonal Prominence. |
| s21nov.pdf | Solutions for Problems issued 21 Nov. 2000. |
| testexam.pdf | Test Problems and Solutions (Fall 1998). |
| qf.pdf | How to Recognize a Quadratic Form. |
| jordan.pdf | Jordan's Normal Form. |
| gilite.pdf | Gargantuan Generalized Inverses. |
| normlite.pdf | Notes on Vector Norms. |
| jensen.pdf" | Jensen's Inequality. |