Topics for Math. H110, Linear Algebra and Matrix Theory, Fall semester 2000

Lectures: Tues. - Thurs. 8 - 9:30 am. in 61 Evans; Discussion section: Wed. 8 - 9 am. in 81 Evans.

Matlab will be used for computational examples. For notes see http://www.cs.berkeley.edu/~wkahan/MathH110

Vector spaces:

Abstract linear spaces, subspaces, dimension, basis
Dual spaces, inner/scalar product, outer product/dyad
Cross-product in Euclidean 3-space

Abstract Linear Maps/Transformations:

Domain, codomain/target-space, kernel/nullspace, range
Sums and products of linear maps, inverses
Representation by matrices dependent upon bases
Change of basis, canonical bases (anticipating later developments)

Elementary row and column reductions to canonical forms

Row echelon form, column echelon form, diagonal form
Rank, equality of row rank and column rank, nullity
Triangular factorizations and variants of Gaussian Elimination, Fredholm's Alternatives

Determinants

Determinant as ratio of volumes, obtainable from triangular factors
Determinantal expansions, Cramer's rule, Jacobi's formula for derivative

Convexity

Convex body as convex hull of points, as intersection of half-spaces
Support planes, separating planes
Linear programming, the Simplex algorithm

Normed linear spaces

Vector norms, triangle inequality, convergence, completeness, compactness
Dual norms, operator/matrix norms, projections
Nearness to singularity, norm of inverse, ill conditioned linear systems
Euclidean and Unitary spaces, orthogonal maps, transpose of matrix
Gram-Schmidt orthogonalization, positive definite matrices, Cholesky factorization
Least Squares, Linearly constrained least squares

Eigenvalues and Eigenvectors

Triangularization by similarity, block triangularization
Characteristic polynomial, Cayley-Hamilton theorem
Jordan's normal form, irreducible invariant subspaces, continuity and derivatives of eigenvalues
Real symmetric matrices, variational derivation of eigenvalues
Singular value decomposition

Applications to … (as time permits)

Positivite matrices, Perron-Frobenius theory, stochastic matrices
Linear differential equations, matrix exponentials
…

Text: The best text would be *Linear Algebra* by P.D. Lax (1997, Wiley) but for the fact that it is a graduate level text more likely to be appreciated after than before this course. And it has no drill problems. Better to buy any text that covers most the topics and is cheap enough to throw away afterwards; then buy Lax's text for a reference.

Prof. W. Kahan