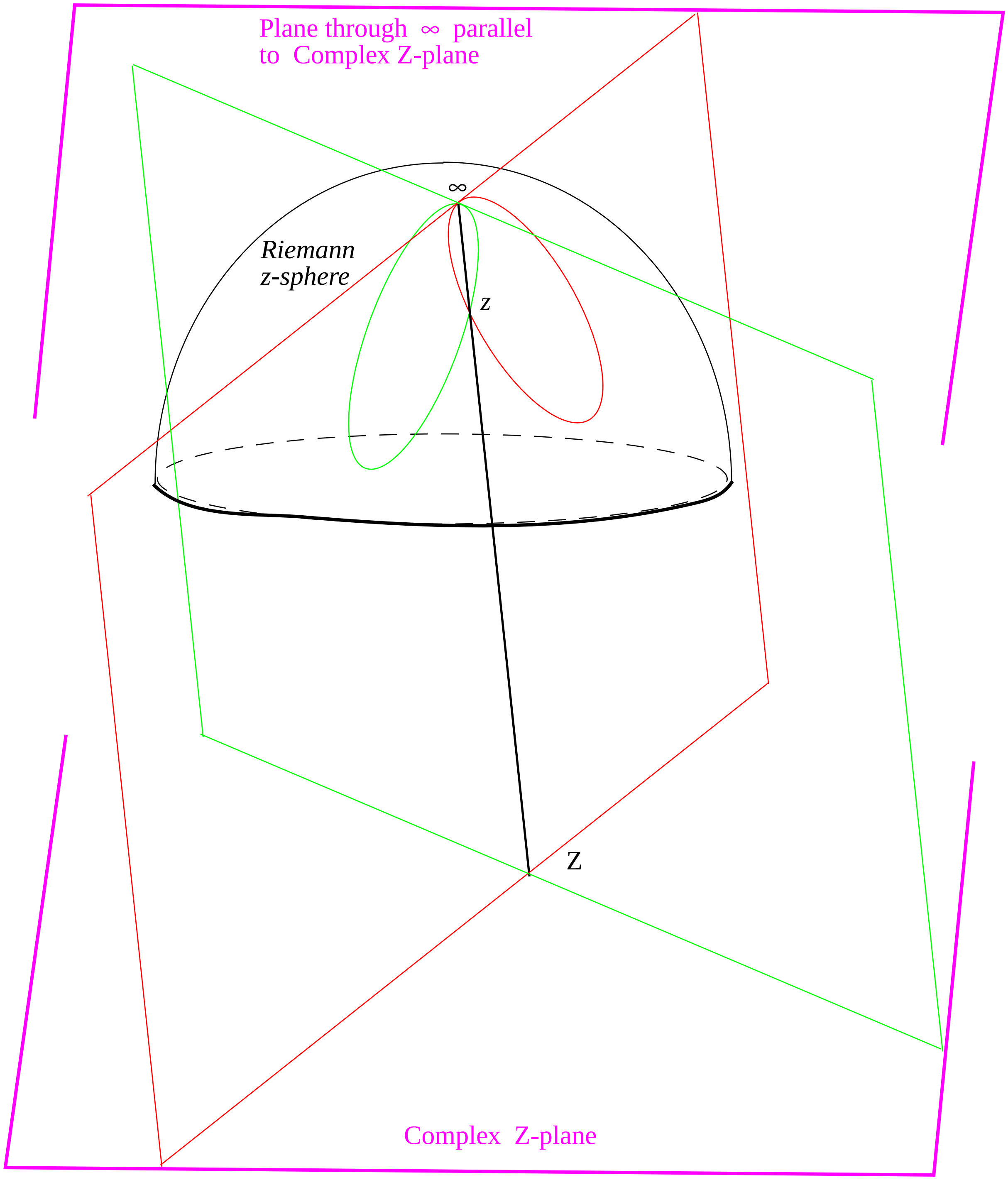
Caratheodory's Proof:



Stereographic projection maps the intersection Z of two lines, one red and one green, in the Complex Z-plane to the intersection z of two circles, one red and one green, in the Riemann z-sphere. The circles are cut from the sphere by two planes, red and green respectively, through the lines in the Z-plane and through ∞ on the sphere. These planes' intersection is the line through Z, z and ∞ . A plane through ∞ parallel to the Z-plane cuts the red and green planes in two lines that intersect at the same angle as do the original lines in the Z-plane, and also at the same angle as do the two circles tangent to them at ∞ . The two circles intersect again at the same angle at z . Therefore Stereographic projection between the Z-plane and the z-sphere preserves curves' angles of intersection.