Semi-separable Linear Systems for $n = 5$

$$\mathcal{A} = \begin{pmatrix} \delta_1 & \alpha_1\gamma_2 & \alpha_1\gamma_3 & \alpha_1\gamma_4 & \alpha_1\gamma_5 \\ \mu_2\nu_1 & \delta_2 & \alpha_2\gamma_3 & \alpha_2\gamma_4 & \alpha_2\gamma_5 \\ \mu_3\nu_1 & \mu_3\nu_2 & \delta_3 & \alpha_3\gamma_4 & \alpha_3\gamma_5 \\ \mu_4\nu_1 & \mu_4\nu_2 & \mu_4\nu_3 & \delta_4 & \alpha_4\gamma_5 \\ \mu_5\nu_1 & \mu_5\nu_2 & \mu_5\nu_3 & \mu_5\nu_4 & \delta_5 \end{pmatrix} .$$

We solve $\mathcal{A}x = b$.

Left Givens rotation on first two rows

$$\mathcal{A} \to \begin{pmatrix} * & * & 0 & 0 & 0 \\ * & * & \hat{\alpha}_2\gamma_3 & \hat{\alpha}_2\gamma_4 & \hat{\alpha}_2\gamma_5 \\ \mu_3\nu_1 & \mu_3\nu_2 & \delta_3 & \alpha_3\gamma_4 & \alpha_3\gamma_5 \\ \mu_4\nu_1 & \mu_4\nu_2 & \mu_4\nu_3 & \delta_4 & \alpha_4\gamma_5 \\ \mu_5\nu_1 & \mu_5\nu_2 & \mu_5\nu_3 & \mu_5\nu_4 & \delta_5 \end{pmatrix} .$$

Also Givens rotation on right hand side.

Right Givens rotation on first two columns

$$\mathcal{A} \rightarrow \begin{pmatrix} * & 0 & 0 & 0 & 0 \\ * & \hat{\delta}_2 & \hat{\alpha}_2\gamma_3 & \hat{\alpha}_2\gamma_4 & \hat{\alpha}_2\gamma_5 \\ \mu_3\hat{\nu}_1 & \mu_3\hat{\nu}_2 & \delta_3 & \alpha_3\gamma_4 & \alpha_3\gamma_5 \\ \mu_4\hat{\nu}_1 & \mu_4\hat{\nu}_2 & \mu_4\nu_3 & \delta_4 & \alpha_4\gamma_5 \\ \mu_5\hat{\nu}_1 & \mu_5\hat{\nu}_2 & \mu_5\nu_3 & \mu_5\nu_4 & \delta_5 \end{pmatrix} .$$

Also Givens rotation on unknown vector x.

We can now solve for the first component in rotated unknown vector.

Define $u_3 = (\mu_3, \cdots, \mu_n)^T$**; and**
b_3 **= last** $n-2$ **components of** b**.**
We now have

$$\begin{pmatrix} \hat{\delta}_2 & \hat{\alpha}_2\gamma_3 & \hat{\alpha}_2\gamma_4 & \hat{\alpha}_2\gamma_5 \\ \mu_3\hat{\nu}_2 & \delta_3 & \alpha_3\gamma_4 & \alpha_3\gamma_5 \\ \mu_4\hat{\nu}_2 & \mu_4\nu_3 & \delta_4 & \alpha_4\gamma_5 \\ \mu_5\hat{\nu}_2 & \mu_5\nu_3 & \mu_5\nu_4 & \delta_5 \end{pmatrix} x_1 = \begin{pmatrix} * \\ b_3 - \rho u_3 \end{pmatrix}$$

for some constant ρ**.**

This is a system similar to the original one, and can be solved similarly.