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Take-Home Test for Math. 128B Solutions due Mon. 8 Mar. 2004

Please be sure your name and student ID# are put on each page of whatever you hand in. The following computations are to be programmed and/or performed by yourself without help from anyone else. You may use MATLAB or any other calculator or computer system. Credit will be awarded less for your results than for your explanation showing that you understand them.

Problem 1:

When evaluated on any computer or calculator, four expressions " (4.0/3.0 – 1.0)·3.0 – 1.0 ", " (10.0/3.0)·3.0 – 10.0 ", " ((10.0/3.0)·3.0 – 5.0) – 5.0 ", and " (2.0/3.0 – 0.5)·3.0 – 0.5 " reveal something about the machine's floating-point arithmetic. Provide examples from at least three different arithmetics, including at least one hand-held calculator, and explain what they reveal. You may use MATLAB 6.x's `system_dependent('setprecision', xx)` on a *Windows* PC to get some of those examples.

Problem 2:

For $x > 0$ the function

$$f(x) := \text{if } x < 1 \text{ then } \arctan\big(\log(1/x)\big)/\arccos(x)^2$$
$$\text{else if } x = 1 \text{ then } 1/2$$
$$\text{else } \arctan\big(\log(x)\big)/\operatorname{arccosh}(x)^2$$

has a smooth graph; in fact, for x sufficiently near 1 this $f(x)$ has a Taylor Series

$$f(1+z) = 1/2 - z/6 + z^2/20 + 124z^3/945 + \dots .$$

Supply a MATLAB program to compute $f(x)$ for $x > 0$ as well as you can using only MATLAB's floating-point (not MAPLE's). How (in)accurate is your program, and why do you think so?

Problem 3:

The three vector norms we use most often are $\|\mathbf{x}\|_\infty := \max_j |x_j|$, $\|\mathbf{x}\|_2 := \sqrt{\left(\sum_j |x_j|^2\right)}$, and $\|\mathbf{x}\|_1 := \sum_j |x_j|$. Exhibit the values of $M_{ij} := \max_{\mathbf{x}\neq\mathbf{o}} \|\mathbf{x}\|_i/\|\mathbf{x}\|_j$ for all six pairs (i, j) with $i \neq j$, and justify them.