QR-Iteration to compute eigenvalues of symmetric matrices:
"Bulge-Chasing" *vs*. Interval Arithmetic:

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 ←→          ←→
↑Wb         Vgx        V«←→       V«         V«         V«         V«
↓qWb        ↑∂◊h       »Vgx       »V«←→      »V«        »V«        »V«
: qWb       ↓xyWb      ↑∂◊h       »Vgx       »V«←→      »V«        »V«
:  qWb        qWb      ↓xyWb      ↑∂◊h       »Vgx       »V«←→      »V«
:   qWb        qWb       qWb      ↓xyWb      ↑∂◊h       »Vgx       »V«
:    qWb        qWb       qWb       qWb      ↓xyWb      ↑∂◊h       »Vg
L……   qW         qW        qW        qW        qW       ↓xyW        ∂◊
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Each QR-Iteration-step consists of a sequence of 2-by-2 rotational similarities applied to consecutive rows and columns. The first of these rotations applied to the first two rows and columns is determined from the last two rows and columns, creating a bulge in the third row and column. Subsequent rotations push the bulge down and off the end of the tridiagonal matrix. All these symmetric tridiagonal mattrices have the same eigenvalues except for rounding errors; those have been proved to have practically inconsequential effects.

The program's inner loop has no tests-and-branches except to detect off-diagonal elements tiny enough to ignore, allowing the matrix to be broken into shorter pieces that will be diagonalized sooner.

QR-Iterations were invented by John Francis in the late 1950s and proved to converge cubically if started close enough to an eigenvalue. When applied to real symmetric or Hermitian matrices reduced to tri- diagonal form, they were proved in the late 1960s to converge at least quadratically and almost always cubically no matter how they were started. W. Kahan's proof used a monotonic convergence link between QR-Iterations and A. Ostrowski's Rayleigh-Ritz inverse iterations.

Bulge-chasing seems to induce big elements to migrate up and small elements down the matrix, after which off-diagonal elements dwindle.

When an off-diagonal element near the middle is too small but not yet negligible, Bulge-Chasing can be Numerically Unstable in the sense that it can produce a sequence of tridiagonal matrices with unpredictable elements utterly different from what would be produced without rounding errors. However, it produces eigenvalues and eigenvectors that are about as accurate as they are determined by the data if computed carrying a few more digits than are trusted in the data. What can be unpredictable is the order in which the eigenvalues will appear; usually smaller ones appear sooner than bigger ones, but not always. Were raw Interval Arithmetic used to compute eigenvalues, they could be condemned as grossly inaccurate though ordinary rounded arithmetic would always produce satisfactory results.