Carnegie Mellon University Instructor: Venkatesan Guruswami
15-252: More GITCS Term: Spring 2021

Problem Set 5

Due date: Friday, March 12 (by midnight EST)

Instructions

- You are allowed to collaborate with one other student taking the class, or do it solo.
- Collaboration is defined as discussion of the lecture material and solution approaches to the problems. Please note that *you are not allowed to share any written material and you must write up solutions on your own.* You must clearly acknowledge your collaborator in the write-up of your solutions.
- Solutions must be submitted on gradescope. Typesetting in LaTeX is recommended but not required. If submitting handwritten work, please make sure it is a legible and polished final draft.
- You should not search for solutions on the web. More generally, you should try and solve the problems without consulting any reference material other than what we cover in class and any provided notes.
- Please start working on the problem set early. Though it is short, the problem(s) might take some time to solve.

Consider the Disjointness function mentioned in class $\text{DISJ} : \{0,1\}^n \times \{0,1\}^n \to \{0,1\}$ defined as

$$\text{DISJ}(x,y) = \begin{cases} 1 & \text{if } x_i y_i = 0 \text{ for all } i = 1,2,\ldots,n \\ 0 & \text{otherwise} \end{cases}$$

1. Prove that the deterministic communication complexity of DISJ equals $n+1$.
2. In a *streaming algorithm*, the input is a data stream presented as a sequence of items which can be examined only once in a single pass (this is similar to how a finite automata accesses its input).

   Suppose we want a streaming algorithm that on input a stream of integers from $\{1,2,\ldots,n\}$ computes the number of distinct elements that occur in the stream. Show that any deterministic streaming algorithm for this task must use $\Omega(n)$ space.