Carnegie Mellon University Instructor: Venkatesan Guruswami
15-252: More GITCS Term: Spring 2021

Problem Set 4

Due date: Friday, March 5 (by midnight EST)

Instructions

- You are allowed to collaborate with one other student taking the class, or do it solo.
- Collaboration is defined as discussion of the lecture material and solution approaches to the problems. Please note that *you are not allowed to share any written material and you must write up solutions on your own.* You must clearly acknowledge your collaborator in the write-up of your solutions.
- Solutions must be submitted on gradescope. Typesetting in LaTeX is recommended but not required. If submitting handwritten work, please make sure it is a legible and polished final draft.
- You should not search for solutions on the web. More generally, you should try and solve the problems without consulting any reference material other than what we cover in class and any provided notes.
- Please start working on the problem set early. Though it is short, the problem(s) might take some time to solve.

(For you understanding only; no need to turn in). The notes posted on the course page show how one can reduce the TM Halting Problem to the problem of telling if a string is incompressible. This reduction is a bit tricky. The reduction in the other direction, however, is quite straightforward. Can you show how assuming that we have a decider for the TM Halting Problem, one can build a decider for telling if an input string is incompressible?

Now to your actual problem.

For each of the following statements, say if it is True or False, and justify your answer.

1. For all large enough even n, there is an incompressible string in $\{0,1\}^n$ which has an equal number of 0s and 1s.
2. Kolmogorov complexity is monotone with respect to prefixes, i.e., for all strings $x, y \in \{0,1\}^*$, $K(x) \le K(xy)$ where xy is the concatenation of x and y.