Carnegie Mellon University Instructor: Venkatesan Guruswami
15-252: More GITCS Term: Spring 2020

Problem Set 8

Due date: Friday, March 20

Instructions

- You are allowed to collaborate with one other student taking the class, or do it solo.
- Collaboration is defined as discussion of the lecture material and solution approaches to the problems. Please note that *you are not allowed to share any written material and you must write up solutions on your own.* You must clearly acknowledge your collaborator in the write-up of your solutions.
- Solutions must be typeset in LaTeX and emailed to the TA.
- You should not search for solutions on the web. More generally, you should try and solve the problems without consulting any reference material other than what we cover in class and any provided notes.
- Please start working on the problem set early. Though it is short, the problem(s) might take some time to solve.

1. Define the predicate $\mathrm{Maj}_3 : \{0,1\}^3 \to \{0,1\}$ where $\mathrm{Maj}_3(a,b,c) = 1$ if and only if at least two of the three input bits a, b, c equal 1.

   Define MAJ-3-SAT to be the variant of 3-SAT where the constraint Maj_3 is applied to triples of *literals*, and the goal is to ascertain if there is an assignment to the variables such that all Maj_3 constraints are satisfied (and to find such an assignment if one exists). For example, the following is an instance of MAJ-3-SAT:

   $$\mathrm{Maj}_3(x_1, x_2, \neg x_3) \wedge \mathrm{Maj}_3(\neg x_1, x_3, \neg x_4) \wedge \mathrm{Maj}_3(x_2, \neg x_4, x_5) \wedge \mathrm{Maj}_3(\neg x_3, x_4, x_5) .$$

   Prove that MAJ-3-SAT can be solved in polynomial time.

2. Suppose that for some $m \geq 3$, $f : \{0,1\}^m \to \{0,1\}$ is a *polymorphism* for the predicate 1-in-3-SAT : $\{0,1\}^3 \to \{0,1\}$ defined by 1-in-3-SAT$(a,b,c) = 1$ if and only if exactly one of the three input bits a, b, c equals 1. (Recall that this means that for every choice of m triples of bits (a_i, b_i, c_i) each with exactly one 1, exactly one of $f(a_1, a_2, \ldots, a_m)$, $f(b_1, b_2, \ldots, b_m)$ and $f(c_1, c_2, \ldots, c_m)$ equals 1.)

   Prove that f must be a dictator function, i.e., there exists some $i \in \{1, 2, \ldots, m\}$ such that $f(x_1, x_2, \ldots, x_m) = x_i$ for every $x \in \{0,1\}^m$.