Evidence for a Flat, Accelerating Universe and the ΛCDM Model (8.942 Term Paper)

Venkatesan Guruswami*

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Abstract

We collect arguments that seem to indicate that we live in a flat Universe which is expanding at an accelerating rate. While we will appeal to some technical aspects of the theory and observational evidences here and there, the presentation for the large part will be at an elementary, intuitive level that can at least be appreciated, if not understood, with minimal background in cosmology. Our aim here is to introduce to a broad audience, recent progress in cosmology which has made it possible to deduce with reasonable certainty, some very fundamental facts and parameters about the geometry of the Universe we all live in.

1 Standard big-bang cosmology

Over the years the relativistic hot big-bang model has emerged as the most widely accepted and essentially the only tenable model of the Universe that is consistent with all the observational data we have. In fact study of this model is now called *standard cosmology*. While there are several observational pillars of the big-bang model, the central one is the *Hubble redshift* of distant galaxies. In the late 1920's Hubble established that the spectra of galaxies at greater distances were systematically redshifted to longer wavelengths (the redshift z is defined by $1 + z = \lambda_{\text{observed}}/\lambda_{\text{emitted}}$). Interpreting the redshift as a Doppler velocity, Hubble's relationship can be written, for very low redshifts $z \ll 1$, as $z \simeq H_0 d/c$, which implies galaxies at distance d are receding away from us at velocities proportional to their distance from us, i.e the Universe is expanding. The factor H_0, called the *Hubble constant*, is the rate of expansion of the Universe at the present epoch. The incontrovertible discovery of the expansion of the Universe naturally supports the concise and beautiful standard cosmological model. We next give a rapid introduction to standard cosmology and the "Friedmann-Robertson-Walker" spacetime and set the stage for the discussion in future sections.

The mathematical description of standard cosmology is based upon the assumption that the Universe is homogeneous and isotropic, and the resulting solution of Einstein's equations of general relativity.[1] The isotropy and homogeneity assumption puts strong restrictions on the spacetime metric of general relativity and leads to the Robertson-Walker (henceforth RW) line element which is given by (see for instance [2, 15]):

$$ds^2 = a^2(\tau)[\,-c^2 d\tau^2 + d\chi^2 + r^2(\chi)(d\theta^2 + \sin^2\theta d\phi^2)] \tag{1}$$

*Laboratory for Computer Science, MIT, Cambridge, MA 01239. Email: venkat@theory.lcs.mit.edu

[1]It is interesting to note that the assumption of isotropy and homogeneity was introduced by Einstein and others to simplify the mathematics; but as it turns out, this is a remarkably accurate description of the Universe averaged over large distances (greater than 100 Mpc or so).

where

$$r(\chi) = \begin{cases} k^{-1/2}\sin(k^{1/2}\chi), & k > 0; \\ (-k)^{-1/2}\sinh[(-k)^{1/2}\chi], & k < 0; \\ \chi, & k = 0. \end{cases} \tag{2}$$

The parameter k is a measure of the curvature of the universe: $k > 0$ implies a positively curved or closed universe, $k < 0$ represents a negatively curved or open universe, while $k = 0$ represents a flat universe. Note that for $k = 0$, except for the scale factor $a(\tau)$, the above line element is just that of flat Minkowski spacetime. In the above equation τ is the conformal time and is not the proper time measured by a comoving observer (which is $t = \int a(\tau)d\tau$). The cosmic scale factor $a(\tau)$ governs the expansion of the Universe and it describes the scaling up of all physical distances in the Universe (separation of galaxies and wavelenghts of photons); one often normalizes the scale factor to equal unity today, so that $a_0 = a(\tau_0) = 1$. The conformal stretching of the wavelengths of photons accounts for the redshift z of light from distant galaxies; this is given by $1 + z = a(\tau_0)/a(\tau_e) = a^{-1}(\tau_e)$ for light emitted by a galaxy at conformal time τ_e. The Hubble parameter at cosmic proper time t is $H(t) = d\ln a/dt = a^{-2}da/d\tau$.

The solution of Einstein's equations $G_{\mu\nu} = 8\pi G T_{\mu\nu}$ for the above metric yields the important Friedmann equation that relates the Hubble parameter to the mass density and the curvature of the Universe:

$$H^2 = \left(\frac{1}{a}\frac{da}{dt}\right)^2 = \frac{8\pi G\rho}{3} - \frac{kc^2}{a^2} \tag{3}$$

Energy Densities: Critical to the geometry of the Universe is the energy density of the various components present in the Universe. The energy density ρ_i of a given component evolves according to $d(\rho_i a^3) = -p_i da^3$ where p_i is the pressure ($p_i \ll \rho_i$ for non-relativistic matter; $p_i = \rho_i/3$ for radiation – we use units where $c = 1$ in this description). The energy density of matter decreases as a^{-3} owing to volume dilution as a result of expansion, while that of radiation decreases as a^{-4} due to an additional a^{-1} factor to account for redshifting. By Equation (3), the curvature term is negligible at high redshift when the Universe was radiation-dominated. Spatial curvature, however, does affect the geometry at low redshift.

It is convenient to define the cosmological density parameter Ω_i for each component in the present epoch by scaling with respect to the critical density $\rho_{\rm crit} \stackrel{\rm def}{=} 3H_0^2/8\pi G$, i.e $\Omega_i = \rho_i/\rho_{\rm crit}$. The total density parameter today, Ω_0, is then given by

$$\Omega_0 \equiv \sum_i \Omega_i \tag{4}$$

and $\rho = \rho_{\rm crit}\Omega_0$. We can now rewrite the curvature constant using the Friedmann Equation as

$$k = \left(\frac{a_o H_0}{c}\right)^2 (\Omega_0 - 1). \tag{5}$$

(Throughout, we will write present values with a subscript 0.) Hence for a Universe which is flat today, we have $\Omega_0 = 1$, while for a subcritical-density Universe ($\Omega_0 < 1$) is negatively curved, and the supercritical-density Universe ($\Omega_0 > 1$) is positively curved.

Identifying and estimating the various components of the total density Ω is a central question in cosmology; this not only gives an inventory of the contents of the Universe, but also, through Equation (5), sheds light on the geometry of the Universe. Equation (5) also beautifully illustrates one of the basic tenets of General Relativity, namely that matter directly governs the spacetime geometry.

Vacuum Energy and Accelerated Expansion. In later sections, we will describe some evidences that very strongly predict the flatness of our Universe. Through Equation (5) this will imply $\Omega_0 = 1$, while we will see in Section 3.2 that our estimates of total matter density Ω_M from its gravitational effects accounts for less than 40% of the critical density. This will lead us to the unmistakable conclusion about the presence of vacuum energy or something similarly mysterious that constitutes the remaining 60% or so of the critical density. Since determinations of matter density take advantage of its enhanced gravity when it clumps in galaxies, clusters or superclusters, vacuum energy, which is by definition spatially smooth, would not have shown up in the matter inventory. The vacuum energy density also remains a constant and unlike matter or radiation density, does not decrease as the Universe expands and the scale factor a grows. It turns out that the presence of vacuum energy predicts an accelerated expansion of the Universe (this is one of the key signatures of the presence of a cosmological constant). Good (but not conclusive) evidence to such an accelerated expansion was recently obtained in 1998 through the survey of high redshift Type Ia supernovae [13], and to many this was the missing piece in the grand cosmological puzzle about the expansion of the Universe and the presence of a cosmological constant.

We would like to end this section by mentioning a couple of other facts, which together with the expansion of the Universe, provide incontrovertible evidence in support of the hot big-bang model. The first of these is the 2.73 K cosmic microwave background radiation (CMB) which is a snapshot of the Universe when it was about $300,000$ years old. The observed CMB spectrum is exquisitely Planckian: any deviations are smaller than 300 parts per million. The only viable explanation of such perfect black-body radiation is the hot, dense conditions that are predicted to exist at early times in the hot big-bang model. Another observational pillar of standard cosmology is big-bang nucleosynthesis (BBN): the abundance pattern of light elements predicted by BBN is consistent with that seen in the most primitive samples of the cosmos.

2 Evidence for a Flat Universe

The standard isotropic cosmology is a very successful framework to explain a variety of observational phenomena, but prior to the early 1980s there were certain questions that had to be avoided. The initial conditions of the big-bang appear to be odd in a number of ways. One of the principal problems, called the *horizon problem*, concerns the observed homogeneity of the Universe, say as observed in the near uniformity of the CMB temperature across the entire sky. Standard cosmology predicts a particle horizon of about 100 Mpc (which subtends an angle of about 1 degree on the sky) for last scattering at $z \sim 1000$ when the CMB radiations were emitted, and hence it is astonishing that the Universe is homogeneous even though there was not enough time for the different causally disconnected parts to "compare notes" over various quantities.

In order to resolve this problem and a few others (one of which was actually the flatness problem, so its not very surprising that this theory actually predicts a flat Universe!), the inflationary theory of early Universe was developed [5] (see also [7]). The story in this theory begins with a brief period of tremendous expansion – a factor of greater than 10^{27} growth in the scale factor in less than 10^{-32} seconds. While the precise details of this inflationary phase are not understood, in most models the exponential expansion is driven by the potential energy of a scalar field initially displaced from the minimum of the potential energy curve. Inflation blows up a really small subhorizon-sized portion of the Universe (which was causally connected before inflation) to a size much bigger than that of the observable Universe today. Since the original subhorizon-sized portion was causally connected, it can be expected to be smooth and homogeneous – including the very small portion of it that is our observable part of the Universe today. Similarly, our Hubble volume is but a small part of the region that inflated, and it thus looks flat, regardless of the initial curvature of the region that inflated.

The cold dark matter plus inflation scenario looks promising and at present is consistent with a large body of observations. The evidence is by no means definitive, but the inflationary theory appears to be a very good model of early Universe phenomena, and it naturally predicts a flat Universe. An even more direct, and almost definitive, observational evidence for flatness is the measurement of CMB anisotropy which we describe next.

Observations of the CMB, a thermal sea of 2.73 K photons, form the cornerstone of modern cosmology. Its very presence provides strong support for the expanding hot big-bang paradigm of cosmology [11]. Small variations in its temperature at the level of 10^{-5} across the sky encode a wealth of cosmological information which can be extracted with appropriate understanding of the anisotropy formation. There seems to be a growing consensus that CMB anisotropy measurements form the best means to determine the curvature of the Universe (and thereby Ω_0). One reason for this belief is that this method is intrinsically geometric — it is a standard ruler on the last-scattering surface and involves straightforward physics at a simpler time [6].

We now attempt a simple description of how this works. In the early universe, when $T \geq 3000$K, the photons are hot enough to ionize hydrogen; the resulting free electrons glue the baryons to the photons by Compton scattering and electromagnetic interactions, and the resulting dynamics involve a single tightly coupled baryon-photon fluid. Primordial density perturbations gravitationally attract this fluid and try to compress it into potential wells of high density, while the photon pressure resists such compressions. This sets up sound waves or acoustic oscillations in the fluid. As the photons cool due to cosmological expansion and the temperature drops below 3000K, atomic hydrogen starts forming and the photons undergo their last scattering off the free electrons. At last scattering, the acoustic oscillations are frozen in leaving peaks for scales that are caught at extrema of their oscillation. Regions of compression and rarefaction at last scattering represent hot and cold spots respectively in the CMB; the oscillation modes frozen at maximum compression or rarefaction lead to the largest temperature anisotropy and are observed as acoustic peaks. The wavelength of the lowest frequency acoustic mode that has reached maximum compression is the standard ruler on the last-scattering surface.

A formal description of the quantitative aspects of CMB anisotropy study is complicated, we shall only attempt to define the relevant notation so that we can (somewhat reasonably) describe the evidence for flatness that is obtained from CMB anisotropy. Further details can be found in [10, 3]. Let us assume a Gaussian random field of primeval gravitational potential (this is for example predicted by inflation). Calculations have shown that the primary CMB anisotropy $\Delta_0(\underline{n})$, defined as the temperature gradient $\Delta T/T$ in the direction $\underline{n}$ of the sky, is linear in the gravitational potential (and its derivatives and integrals). Thus, the anisotropy at any point in the sky is a linear combination of Gaussian-distributed random numbers and is hence itself a Gaussian random field *on the sphere*. In order to characterize its statistical properties, therefore, we only need to calculate its *angular power spectrum* (which is the spherical analogue of the conventional power spectrum $P_\phi(k)$ defined for modes $\phi(\underline{k})$ of potential fluctuations).

The angular power spectrum C_l is parameterized by an integer l which corresponds to the various modes in the Fourier decomposition of the acoustic oscillations. One expands the temperature anisotropy in the orthonormal basis appropriate for a sphere (namely the spherical harmonics Y_{lm}):

$$\Delta_0(\underline{n}) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} a_{lm} Y_{lm}(\underline{n}).$$

The angular power spectrum C_l is now defined in terms of the covariance of the various multipole amplitudes a_{lm}:

$$\langle a_{lm} a^*_{l'm'} \rangle \stackrel{\text{def}}{=} C_l \delta_{ll'} \delta_{mm'}.$$

It turns out that for a *scale-invariant spectrum*, $P_\phi(k) \propto k^{-3}$, for the primeval potential fluctuations, one can compute C_l exactly and one gets $C_l \propto 1/[l(l+1)]$, corresponding to equal power per logarithmic interval of angular degree l, or in other words a "flat spectrum" on the sphere. Plots of the CMB power spectrum, therefore, always show $l(l+1)C_l$ Vs l.

The position of the first acoustic peak (i.e a peak in the $l(l+1)C_l$ Vs l plot) depends upon Ω_0, and is $l_{\text{peak}} \simeq 200/\sqrt{\Omega_0}$ (see for instance [16]). One can thus deduce a good estimate for Ω_0 from anisotropy measurements that can pinpoint the location of the first acoustic peak. CMB anisotropy measurements now cover three orders of magnitudes in multipoles l and are from more than twenty experiments. Cosmic Background Explorer (COBE) is the most precise, and covers multipoles l in the range 2-20. The other measurements come from balloon-borne, Antarctica-based and ground-based experiments using both low-frequency (< 100 Ghz) HEMT receivers and high-frequency (> 100 Ghz) bolometers. All these measurements taken together are beginning to define the position of the first acoustic peak at a value of about 200, and this evidence was also strongly supported by the recent study of Miller et. al [8]. The position of the first acoustic peak is consistent with the theoretical prediction for a flat Universe ($\Omega_0 = 1$), and the analyses of all the extant data put together seem to pin-point $\Omega_0 \sim 1 \pm 0.2$. While it is certainly too early to jump to definite conclusions, a strong case is developing for a flat Universe and a lot more data is on the way. The issue will be ultimately settled, one hopes, by NASA's MAP (which will be launched late 2000) and ESA's Planck (launch 2007) satellites which will map the entire CMB sky with 30 times the resolution of COBE.

3 Presence of a Cosmological Constant?

Considering that we today live in a Universe with negligible radiation, the total energy density Ω_0 has two components in our models of the Universe (referred to as cold dark matter models): Ω_M which is the density of all matter (luminous matter, baryons and *cold dark matter*), and Ω_Λ which is the vacuum energy density (if at all any such mysterious form of energy is present). Let us assume that the evidence discussed in the previous section is definitive enough so that we can work with reasonable conviction with the model where $\Omega_0 = 1$. In this section we will look at two models: one the SCDM (standard cold dark matter) model with $\Omega_m = 1$, and the other the ΛCDM model with non-zero cosmological constant but satisfying $\Omega_m + \Omega_\Lambda = 1$. Despite the appealing simplicity of the former model, it will turn out to be inconsistent with observations, leaving ΛCDM as the most viable CDM model we have today and making it increasingly popular as the best-fit model of the Universe as a flood of observational data flow in.

3.1 The SCDM Model: Appealing but untenable!

For the SCDM model (also referred to as an Einstein-de Sitter Universe), solving the Friedmann Equation (3) (with $K = 0$) gives the scale factor as $a(t) = (3H_0t/2)^{2/3}$ where $H_0 = 100h$ km s^{-1} Mpc^{-1} is the current Hubble constant. Thus for the SCDM model the product of the Hubble constant and time back to the big-bang, H_0t_0, equals 2/3. It turns out (see for instance [17]) that $\Omega_M = 1$ is possible only if $h \sim 0.6$ and $t_0 \sim 10$ Gyr. This estimate of the age t_0 is the Universe is, however, way too low. Chaboyer *et al.* [4] date the oldest globular stars at 11.5 ± 1.3 Gyr; to obtain an estimate of the age of the Universe, we must add another 1-2 Gyr to account for the time to the formation of the oldest globular clusters. Recent type Ia supernova data by Reiss *et al.* [13] yield an expansion age for the Universe of 14.0 ± 1.5 Gyr, including an estimate of systematic errors. These data pretty much rule out a $\Omega_M = 1$ Universe.

3.2 Determination of Ω_M

It turns out that one can put more stringent upper bounds on Ω_M. The amount of matter known to be present from its gravitational effects indicate $\Omega_M = 0.35 \pm 0.07$ (the error flag can interpreted as indicating a 95% certainty that Ω_M is between 0.2 and 0.5). Moreover, much of even this matter is *dark*; it has been known since the pioneering work of Fritz Zwicky and Vera Rubin that there is far too little material in the form of stars and related material to hold galaxies and clusters together. We can now say with reasonable confidence that stars and other luminous material form a tiny fraction of the critical density, $\Omega_{\text{lum}} \simeq 0.004$. The theory of big-bang nucleosynthesis and the recently measured primeval abundance of deuterium (see for instance Peacock's book [10]) pin down the baryon density very precisely: $\Omega_B = (0.02 \pm 0.002)h^{-2} \simeq 0.05$. This indicates that the bulk of the matter density in the Universe comes from non-baryonic dark matter. What exactly constitutes this mysterious dark matter is an entire area of cosmology and particle physics by itself which we will not get into, but we just mention that there is no definitive theory (or consensus) yet on the composition of the dark matter. Our concern will be on estimating the total matter density so that we can get a handle on Ω_M, and we will be oblivious to the composition of the matter density itself.

A variety of methods for determining the total amount of matter all seem to converge on $\Omega_M \sim 1/3$; the most reliable of them appears to be measurements of the masses of clusters of galaxies, and we therefore describe this briefly below.

Rich clusters are relatively infrequent objects; less than a tenth of the galaxies are found in a rich cluster that formed from density perturbations of comoving size around 10 Mpc. But these clusters, rare though they are, gather together material from such a large region of space that they can provide a "fair sample" of matter in the Universe. Using these clusters, the precise BBN baryon density can be used to infer the total matter density [19]. The baryons in the clusters mostly reside in hot, x-ray emitting intracluster gas and not in the galaxies themselves, so one just needs to determine the total gas-to-mass ratio f_{gas} in order to get the baryon to total mass ratio.

Mohr *et al.* [9] have compiled the gas to total mass ratio determined from x-ray measurements for a sample of 45 clusters and they find $f_{\text{gas}} = (0.075 \pm 0.002)h^{-3/2}$. Assuming that these clusters give a fair sample of the Universe we can infer the mean matter density in the Universe as:

$$\Omega_M = \Omega_B / f_{\text{gas}} \simeq 0.4 \tag{6}$$

Turner [16] gives a few arguments about why the above is one of the most reliable and precise determination of the matter density. We now proceed taking for a fact that $\Omega_M \simeq 0.4$ to see what consequence this has for the Universe when put together with $\Omega_0 \simeq 1$.

3.3 Existence of a Cosmological Constant

The two apparently contradictory statements $\Omega_0 \sim 1$ and $\Omega_M \sim 0.4$ leads us to seriously consider the possibility that the missing 60% or so of the critical density exists in the form of vacuum energy (cosmological constant) or something even more interesting with similar properties. The basic property we need is that of a smooth distribution which does not clump and hence cannot be detected in our matter inventory. The cosmological constant is the simplest possibility and has equation of state $p = -\rho$ (in units where $c = 1$). There are other possibilities, though, for such smooth energy, and their bulk equation of state may be parameterized by $w = p/\rho$. This implies that the energy density evolves as $\rho \propto a^{-n}$ where $n = 3(1 + w)$. The large-scale structure of the Universe as we see today that arose from density perturbations (inferred from CMB anisotropy measurements) imply that the Universe had to be matter-dominated from the epoch of matter-radiation equality until very recently (i.e the cosmological constant dominated phase of the Universe did not begin until fairly recently). This implies that the "smooth energy"' component did not interfere with structure formation, and hence must have been less important in the past than it is today. This in turn implies than we must have $n < 3$, or in other words $w < 0$; and the more negative w is, the faster the "smooth energy" component gets out of the way of structure formation. A more careful consideration [18] of the growth of the structure implies that w must in fact be less than about $-1/3$.

There is also a strong constraint on w that is posed by measurements of the age of the Universe t_0 and the present day Hubble constant H_0. The product $H_0 t_0$ depends on the equation of state of the Universe and $H_0 t_0$ increases with decreasing w (for example $H_0 t_0 = 1/2$ for a radiation dominated Universe where $w = 1/3$ while $H_0 t_0 = 2/3$ for a matter-dominated Universe where $w = 0$). Taking $t_0 = 14 \pm 1.5$ Gyr and $H_0 = 65$ km s^{-1} Mpc^{-1} implies $H_0 t_0 = 0.93 \pm 0.13$ and it can be shown that this is preferred by $w < -1/2$.

To summarize, in order to reconcile $\Omega_0 \sim 1$ and $\Omega_M \sim 0.4$ along with some other cosmological considerations necessitates the existence of a "smooth energy" component with bulk pressure p more negative than $-\rho/2$. The simplest example of such is the vacuum energy (Einstein's infamous cosmological constant) which has $w = -1$. We will see in the next section that a key signature of such an energy component is the prediction of an accelerating Universe and we will also touch upon recent evidences which point that this indeed might be the case.

4 Case for an accelerating Universe

We will argue in this section that the presence of a "smooth energy" component with bulk equation of state $p = w\rho$ (where $-1 \leq w < 0$; $w = -1$ will correspond to vacuum energy or cosmological constant) will lead to an accelerating expansion of the Universe if $w < -5/9$. Before we formally see this let us give a semi-qualitative argument to demonstrate this when $w = -1$. Indeed in this case, by the Friedmann equation we have

$$H(a) = \frac{1}{a}\frac{da}{dt} = H_0\sqrt{\Omega(a)} = H_0\sqrt{\Omega_{m0}a^{-3} + \Omega_\Lambda} \tag{7}$$

In this case the cosmological constant becomes dominant when the size of the Universe is $[\Omega_{m0}/(1-\Omega_{m0})]^{1/3}$ times its present size (we assume $\Omega_\Lambda = 1 - \Omega_{m0} > \Omega_{m0}$), and thereafter the scale factor grows exponentially, and consequently the expansion happens at an accelerating rate.

The change in the rate of the Hubble expansion is classically measured by the *deceleration parameter* (Sandage, 1961 [14]) q_0, and it quantifies the present *slowing* of expansion due to gravity. Historically the oldest approach to determining the total mass-energy density was through the deceleration parameter [1, 14]. It is defined as:

$$q_0 = -\frac{((d^2a/dt^2)/a)_0}{H_0^2}. \tag{8}$$

For a component with equation of state $p = w\rho$, the first law of thermodynamics yields $\rho \propto a^{-3(1+w)}$.

Using this in the Friedmann equation yields

$$\frac{da}{dt} = H_0 a \sqrt{\sum_i \Omega_i a^{-3(1+w_i)}}, \tag{9}$$

where Ω_i is the density parameter for component i whose bulk equation of state is $p_i = w_i \rho_i$ (for example for baryons $w_i = 0$; for radiation $w_i = 1/3$; and $w_i = -1$ for vacuum energy). Differentiating the above with respect to t and after a few simple algebraic manipulations we get an expression for the deceleration parameter today in terms of the density parameters:

$$q_0 = \frac{1}{2}\Omega_0 + \frac{3}{2}\sum_i \Omega_i w_i \tag{10}$$

where Ω_0 is the total density parameter.

For a ΛCDM model with $\Omega_0 = 1$ and $\Omega_M = 0.4$ and with the equation of state of the "smooth energy" component Ω_X being $p_X = w_X \rho_X$, we have $q_0 = 1/2 + 3/2 \times 0.6 w_X$, so that $q_0 < 0$ for $w_X < -\frac{5}{9}$. Hence $w < -5/9$ leads to a negative deceleration parameter or in other words to accelerated expansion. The kind of energy that is required to pull cosmology together and to explain the flatness of the Universe therefore ends up predicting accelerated expansion.

Reiss *et al* [13], in an important recent work, use spectral and photometric observations of high and low redshift Type Ia supernovae (SNe Ia), to give evidence in favor of an accelerating Universe. Presenting this evidence in any reasonable detail is difficult and beyond the scope of this article, but we outline the high level idea. The *luminosity distance* of a supernova at redshift z is defined by $D_L(z) = (L/4\pi S)^{1/2}$ where L and S are the SN's intrinsic luminosity and observed flux respectively. Prediction of the cosmological parameters (like Ω_M, Ω_Λ and q_0) can be obtained with good statistical confidence once good estimates of z and D_L are obtained for several supernovae in a broad enough range of redshifts. (In the Reiss *et al* study the high z supernovae lie in the redshift range $0.16 \le z \le 0.62$; getting statistically significant data for even larger redshift is a challenge for the future but even the existing data suffices to constrain the cosmological parameters quite remarkably.)

A critical component of the work of Reiss *et al* is the determination of the luminosity distances of the observed SNe Ia. One way to do this is the so called "Multicolor light curve shape method" where the distance estimates are obtained by employing the observed correlation between light curve shape, luminosity and color. Details on this can be found in [12, 13].

In FRW cosmologies, the luminosity distance at a given redshift, z, is a function of the cosmological parameters. Limiting our consideration of these parameters (for sake of simplicity) to H_0, Ω_M and Ω_Λ, say with the added assumption of flatness $\Omega_M + \Omega_\Lambda$ (this is not necessary, but let us make this assumption to keep things simple, and since we so vehemently campaigned for the cause of a flat Universe in Section 2 anyway), the luminosity distance is

$$D_L(z) = \frac{c}{H_0}(1+z)\int_0^z \frac{H_0 dz'}{H(z')} = \frac{c}{H_0}(1+z)\int_0^z dz'[\Omega_\Lambda + \Omega_M(1+z')^3]^{-1/2}. \tag{11}$$

Hence $z - D_L(z)$ data for a sizeable sample of SNe Ia over a range of redshifts constrains, through the above equation, the possible values that $H_0, \Omega_M, \Omega_\Lambda$ can take. This is what is done in [13] and putting all the existing data together gives a robust (99.5% confidence level) indication for a negative sign of the deceleration parameter q_0 (and hence an accelerating Universe), and a more uncertain estimate for its value $q_0 = -1.0 \pm 0.4$. More SNe Ia at redshifts greater than 0.5 will be needed to yield a more robust indication for the value of q_0. The complete spectroscopic SNe Ia data with the assumption $\Omega_M \geq 0$ also predict a positive cosmological constant with 99.7% confidence. The dynamical age of the Universe is also estimated by Reiss *et. al* to be 14.2 ± 1.7 Gyr including systematic uncertainties in the current Cepheid distance scale. They also estimate the likely effect of several sources of systematic error, including sample selection bias, evolution, extinction, local perturbations in the expansion rate and sample contamination, and indicate that currently none of these effects appear to reconcile all the data with $\Omega_\Lambda = 0$ and $q_0 \geq 0$. Thus, while the arguments in support of an accelerating Universe are not yet beyond dispute, there is a very strong indication for this actually being the case.

5 Summary and Conclusions

The hot-big bang cosmological model, extended with inflation + cold dark matter, offers a convincing and elegant explanation of the history of the Universe to within a small fraction of a second after its beginning. The inflationary prediction of the flatness of the Universe has been supported by observational data from an invaluable resource: Cosmic Microwave Background Anisotropy, and there is now a very strong consensus on the flatness of our observable Universe. Flatness in turn has an important consequence for the composition of the energy density of the Universe (since matter determines the geometry in general relativity) — it implies that the total energy density of the Universe must equal the critical density required for flatness. This, together with the reasonably accurate guess we have on the matter density $\Omega_M \simeq 0.4$ through its observed gravitational effects, leads one to the unmistakable conclusion that there must be a "smooth energy" component, like vacuum energy (cosmological constant), which constitutes the remaining 60% or so of the critical density. This leads to the ΛCDM model of the Universe as being the most tenable cold dark matter model of the Universe. The ΛCDM model, because of the presence of a smooth energy component with negative pressure, predicts an accelerated expansion of the Universe. Recent study by Reiss *et al* [13] uses observations of high redshift supernovae to provide strong evidence that this is indeed the case. While it remains to be conclusively seen whether or not the expansion is accelerating, it is certainly the case that our knowledge of the Universe is accelerating, driven by new observational results.

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