Photons as qubits

Cronin's Lecture #8

(1)

<u>Photons as Qubits</u> → Many ways to use photons as qubits!
First, think about <u>what is a photon?</u>

Will discuss is Polarization

<u>Consider Classical E & M</u> : <u>Maxwell's Eq'ns</u> :
In a dielectric (insulating! non-magnetic) material (includes vacuum!)

$$\Rightarrow \vec{\nabla}\cdot\epsilon\vec{E} = 0, \quad \vec{D} = \epsilon\vec{E}, \quad \epsilon = \text{electric polarization (dielectric constant)}$$

$$\vec{\nabla}\times\vec{E} = -\frac{1}{c}\frac{\partial\vec{B}}{\partial t} \qquad \epsilon = 1 + 4\pi\frac{P}{E}, \quad P = \frac{\text{electric dipole moment}}{\text{volume}}$$

$$\vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\times\vec{B} = \frac{\epsilon}{c}\frac{\partial\vec{E}}{\partial t}$$

Photon = Q. excitation of a field

Behavior of photon closely linked to Classical $\vec{E}$ & $\vec{B}$ fields

⇒ Solution to these equations gives traveling E&M wave:

$$\vec{E}(\vec{r},t) = \vec{E}_0\cos(\vec{k}\cdot\vec{r} - \omega t)$$

Also have ⊥ $\vec{B}$-field : $\vec{B}(\vec{r},t) = \vec{B}_0\cos(\vec{k}\cdot\vec{r} - \omega t)$

<u>Some properties</u> : $\vec{E}_0 \perp \vec{B}_0 \perp \vec{k}$, $\vec{E}_0\times\vec{B}_0 \parallel \vec{k}$

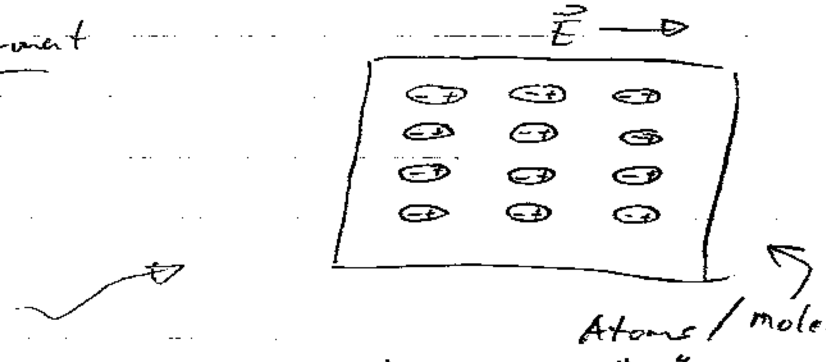
In cgs units ⇒ $|\vec{B}| = |\vec{E}|$, so let's ignore $\vec{B}$-field since if we know $\vec{E}$ ⇒ we know $\vec{B}$.
Also, in a material, it's the $\vec{E}$-field that couples more strongly to electrons ($\vec{F} = q\vec{E}$), so $\vec{E}$-field is more important.

⇒ <u>Traveling Wave</u>! <u>V_{ph} = Phase velocity</u> $= \frac{\omega}{k} = \frac{c}{\sqrt{\epsilon}}$, $k = \frac{\omega}{c}\sqrt{\epsilon}$
$c = 3\times10^{10}$ cm/sec, ϵ = polarizability of medium
<u>More Familiar</u> : "n" = index of refraction, $n = \sqrt{\epsilon}$

$V_{light} = \frac{c}{n}$, $n = \sqrt{\epsilon}$ ($\sqrt{\epsilon} = 1$ for vacuum)

ϵ depends on material properties : $\epsilon = 1 + \frac{4\pi |\vec{P}|}{|\vec{E}|}$

$\vec{P} = \frac{\text{dipole moment}}{\text{volume}}$



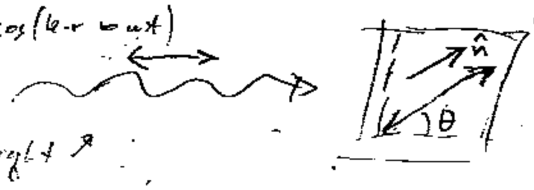
The more they polarize ⇒ the bigger "ϵ" is and the slower light travels! Note : Some materials might polarize more in one direction than another. ⇒ Speed of light can be different for different polarizations of light.

$\boxed{\text{Defn}}$: Polarization of light : The direction that $\vec{E}$ points.

$$\vec{E} = \vec{E}_0 \cos(\vec{k}\cdot\vec{x} - \omega t) \Rightarrow \text{Pol.} \parallel \vec{E}_0 \quad (\perp \vec{k})$$

Polarizer: This is a material that only allows light to pass with $\vec{E}$ polarized in a particular direction:

in: $\vec{E}_0\cos(k\cdot r - \omega t)$



out: $E_0'\cos(ky - \omega t)$

$|\vec{E}_0'| = \vec{E}_0 \cdot \hat{n} = E_0\cos\theta$

$$\frac{\vec{E}_0'}{E_0'} = \hat{n}$$

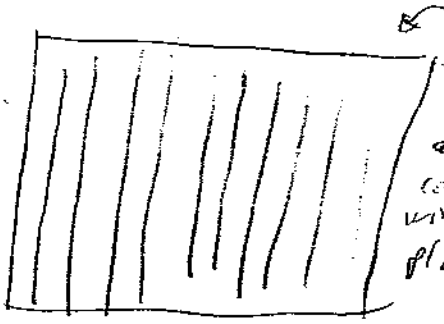
[light propagation direction must be out of page]

a) Light emerging must have polarization of polarizer

b) Amplitude of $\vec{E}$ is reduced by $\cos\theta$

How make polarizer?

Component of $\vec{E}$-field $\parallel$ to lines causes charge to flow and energy is absorbed. Component $\perp$ to lines is not absorbed, passes freely

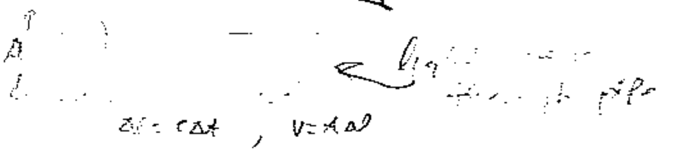


Put lots of aligned conducting wires on a plate of glass

Energy of light: Classically a light wave fills a volume and has an energy density associated with it: $\rho = \frac{\text{Energy}}{\text{Volume}} = \frac{|E|^2}{8\pi}$

$\Rightarrow$ Energy of light wave in a volume of space (V) is

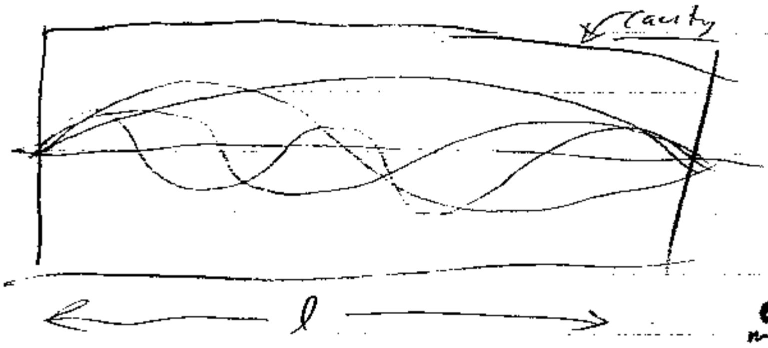
$$U_{vol} = \rho V = \frac{|E|^2}{8\pi} V$$



Fact: Light actually travels in packets of energy called photons. Each photon has an $\vec{E}$-field and a frequency (ω) associated w/ it. $U_{photon} = \hbar\omega$

Why is light quantized? Different ways to approach this. Full treatment is Q.E.D., see QED book by Feynman. I will not derive this.

One way to think of it is that if I have light in a cavity ⇒ The space inside the cavity can be thought of as a bunch of simple harmonic oscillators with different frequencies. Each frequency is a mode of the cavity:



Different modes: $\lambda = \frac{\ell}{n}$, $n = \text{int}$

$\omega_n = \frac{2\pi c}{\lambda} = \frac{2\pi n c}{\ell}$

⇒ photon is a quantized vibration of SHO having freq $= \omega$ (in mode "ω_n")

Consequence Of Quantization:

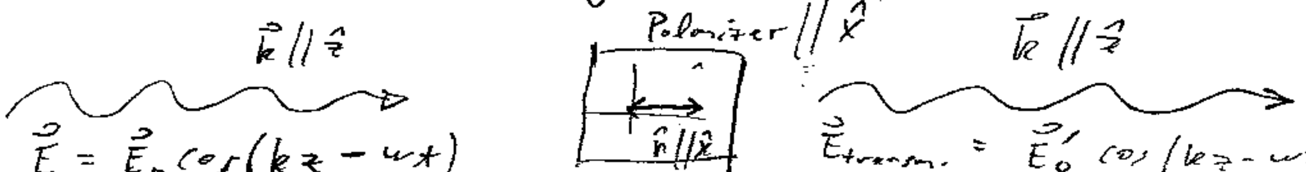
1) # of photons in a light wave depends on magnitude of $\vec{E}$-field: Consider $\vec{E} = \vec{E}_0 \cos(\vec{k}\cdot\vec{r} - \omega t)$ in volume V

$$\Rightarrow U_T = \frac{E_0^2}{8\pi} V = N\hbar\omega, \quad N = \#\text{ photons}$$

$$\Rightarrow \boxed{N = \frac{E_0^2 V}{8\pi\hbar\omega}}$$

$= \frac{E_{\text{photon}}^2}{8\pi} V = \hbar\omega$, $E_{ph} = \sqrt{\frac{\hbar\omega 8\pi}{V}}$

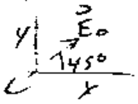
2) Probabilistic Behavior : Suppose we shine light on a polarizer ⇒ How do we interpret behavior in terms of photons?



$\vec{E}_{incident} = \vec{E}_0 \cos(kz - \omega t)$

$\vec{E}_{transm.} = \vec{E}_0' \cos(kz - \omega t)$

$\vec{E}_0 = E_{0x}\hat{x} + E_{0y}\hat{y}$, $E_{0x} = E_{0y}$



$\vec{E}_0' = E_0 \cos 45° = E_{0x}$
$= E_0/\sqrt{2}$

$\vec{E}_0' \parallel \hat{x}$

$\boxed{\text{region } \# 1}$

Volume V $[= (\text{Area}) \times (c\Delta t)$ for pulse of length Δt in pipe w/ cross section $A]$

$\boxed{\text{region } \# 2}$
Volume V

In region # 1 have N photons, all identical with identical $\vec{E}_0$ and ω.

$N = \frac{1}{8\pi}\frac{E_0^2 V}{\hbar\omega}$

In region # 2 have N' photons, all identical, w/ $\vec{E}_0'$ and ω

$\Rightarrow N' = \frac{1}{8\pi}\frac{E_0'^2 V}{\hbar\omega} = \frac{1}{8\pi}\frac{(E_0/\sqrt{2})^2 V}{\hbar\omega} = \frac{1}{2}\left(\frac{1}{8\pi}\frac{E_0^2 V}{\hbar\omega}\right)$

$\Rightarrow \boxed{\frac{N'}{N} = 1/2}$

BUT, this is STRANGE since all the photons in region #1 are IDENTICAL. Why do only half get through? ⇒ Transmission MUST be a probabilistic process ⇒ We are led to a probabilistic / Q.M. interpretation?

Probability for transmission $= \dfrac{|E_0'|^2}{|E_0|^2}$

<u>But</u> $\vec{E}_0 = E_{0x}\hat{x} + E_{0y}\hat{y} \Rightarrow |E_0|^2 = E_{0x}^2 + E_{0y}^2$

$E_0' = E_{0x}^2$

$$\Rightarrow \text{Prob.} = \frac{E_{0x}^2}{E_{0x}^2 + E_{0y}^2}$$

$\Rightarrow$ Components of $\vec{E}$-field vector in diff. directions act as probability amplitudes! like $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$, Prob. $|0\rangle = \frac{\alpha^2}{\alpha^2+\beta^2}$ etc.

$\Rightarrow$ Can think of $\vec{E}$-field polarization as a Q.M. observable!

$\Rightarrow$ Photon w/ $\vec{E} = E_{0x}\hat{x} + E_{0y}\hat{y}$

Can be thought of as living in a state $|\psi\rangle$

where $|\psi\rangle = \begin{pmatrix} \psi_x \\ \psi_y \end{pmatrix}$ $\leftarrow$ x-polarized component, $\leftarrow$ y-polarized component

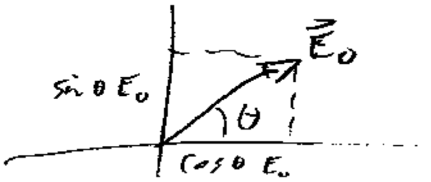
For 1 photon living in a volume "V" $\Rightarrow$

$|\psi_x|^2 + |\psi_y|^2 = 1$, $\psi_x = \sqrt{\frac{V}{8\pi\hbar\omega}}\, E_{x_{phot}}$ ($\vec{E}\cos\theta$) $\psi_y = \sqrt{\frac{V}{8\pi\hbar\omega}}\, E_{y_{photon}}$ ($E\sin\theta$)

$$\Rightarrow |\psi_x|^2 + |\psi_y|^2 = \underbrace{\frac{V}{8\pi\hbar\omega}E_x^2}_{\cos\theta} + \underbrace{\frac{V}{8\pi\hbar\omega}E_y^2}_{\sin\theta} = \left(\frac{E^2}{8\pi}\cdot V\right)\left(\frac{1}{\hbar\omega}\right) = 1$$

The $\hat{x}$ & $\hat{y}$ polarization vectors form a basis!

Suppose we have $\vec{E}_0 = E_{0x}\hat{x} + E_{y}\hat{y}$



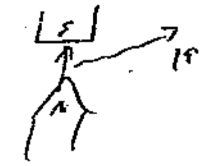
$$\Rightarrow |\psi\rangle = \begin{pmatrix}\cos\theta \\ \sin\theta\end{pmatrix} = \cos\theta|x\rangle + \sin\theta|y\rangle$$

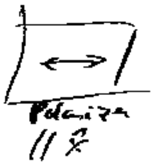
$\Rightarrow$ Polarization of the photon is our qubit variable: $|0\rangle = |x\rangle$, $|1\rangle = |y\rangle$

$$|\psi\rangle = \underset{\substack{\uparrow \\ \vec{E}_0 \text{ component} \\ \text{along } \hat{x}}}{\alpha|0\rangle} + \underset{\substack{\uparrow \\ \vec{E}_0 \text{ component} \\ \text{along } y}}{\beta|1\rangle}$$

How do we measure & transform this qubit?

<u>Similar to Spin</u>: Polarizer plays the same role as S-G device:

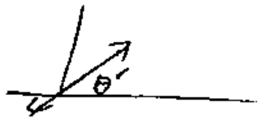
$|spin\rangle = \alpha|\uparrow\rangle + \beta|\downarrow\rangle \rightsquigarrow$  $|\uparrow\rangle$ w/ Prob. $|\alpha|^2$

$|photon\rangle = \alpha|x\rangle + \beta|y\rangle \rightsquigarrow$ 

$\rightsquigarrow |x\rangle$ w/ prob. $|\alpha|^2$

Consider Bloch sphere : $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$
$= \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle$

$\Rightarrow$ Polarizer at angle θ' wrt $\hat{x}$ leads to



$$|\psi\rangle_{photon} = \cos\theta'|x\rangle + \sin\theta'|y\rangle$$

$$= \cos\frac{\theta}{2}|0\rangle + \sin\frac{\theta}{2}e^{i\phi}|1\rangle$$

if we identify $\theta = 2\theta'$, $\phi = 0$ here

$\Rightarrow$ How do we change ϕ?? Relative phase?

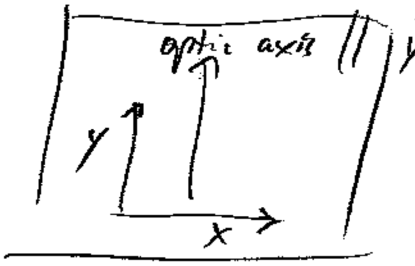
$\Rightarrow$ Must change relative phase between $\vec{E}_{0x}$ and $\vec{E}_{0y}$ for traveling photon. $\Rightarrow$ How do this?

Answer: Find material where index of refraction is different in $\hat{x}$ & $\hat{y}$ directions!

$\Rightarrow$ Velocity is different for $\hat{x}$ & $\hat{y}$ components!

This can be done by finding ~~an~~ an anisotropic Xtal w/ different polarizability in $\hat{x}$ & $\hat{y}$ directions.

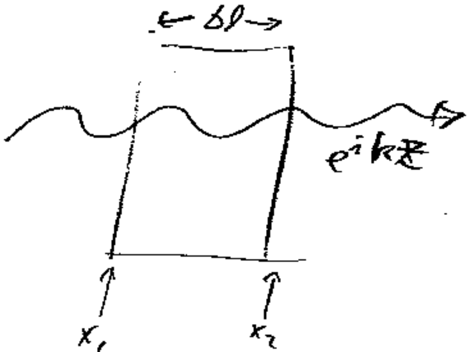
Calcite:



$\Rightarrow \dfrac{n_y}{n_x} \approx 0.9$

Light polarized $\parallel \hat{y}$ goes $\approx 10\%$ faster than light pol. $\parallel \hat{x}$!

If xtal has length Δl $\Rightarrow$ phase difference for light passing through is:



$$\Delta\phi = k\Delta l$$

e^{ikz_1} $\quad e^{ikz_2}$ $\Rightarrow \Delta\phi = k(z_2 - z_1) = k\Delta l$

$\Rightarrow$ For light w/ $\vec{E} \parallel \hat{y} \Rightarrow k_y = \frac{\omega}{c} n_y$

For light w/ $\vec{E} \parallel \hat{x} \Rightarrow k_x = \frac{\omega}{c} n_x$

$\Rightarrow$ Phase change through xtal for $\vec{E}_x$ component $=$
$\Delta\phi_x = \frac{\omega}{c} n_x \Delta l$

Phase change for $\vec{E}_y$ component $= \Delta\phi_y = \frac{\omega}{c} n_y \Delta l$

So, if $|\psi\rangle_{in} = \cos\frac{\theta}{2}|x\rangle + \sin\frac{\theta}{2}|y\rangle$

$$\Rightarrow |\psi\rangle_{out} = \cos\tfrac{\theta}{2}\, e^{i\frac{\omega}{c} n_x \Delta l}|x\rangle + \sin\tfrac{\theta}{2}\, e^{i\frac{\omega}{c} n_y \Delta l}|y\rangle$$

$\Rightarrow$ On Bloch sphere $\Rightarrow$ need $|x\rangle$ amplitude ved, $\Rightarrow$
mult. by phase factor

$\Rightarrow |\psi\rangle_{out} \to \cos\frac{\theta}{2}|x\rangle + e^{i\frac{\omega}{c}\Delta l(n_y - n_x)} \sin\frac{\theta}{2}|y\rangle$

$\Rightarrow$ "ϕ" on Bloch sphere is $\frac{\omega}{c}\Delta l(n_y - n_x)$

$\Rightarrow$ "Xtal" plays same role that $\vec{B}$-field did for spin!!!