1 Photons

Photons are pretty versatile, and there are may ways to use photons as qubits! First, let's think about what a photon is.

Consider classical electricity and magnetism, which are governed by Maxwell's equations. Let's have a look at Maxwell's equations in an insulating, non-magnetic dielectric material:

$$\nabla \cdot \varepsilon \vec{E} = 0 \qquad \nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t} \qquad \nabla \cdot \vec{B} = 0 \qquad \nabla \times \vec{B} = \frac{\varepsilon}{c}\frac{\partial \vec{E}}{\partial t} \tag{1}$$

For a dielectric material, ε is the dielectic constant which determines the polarizability of the material.

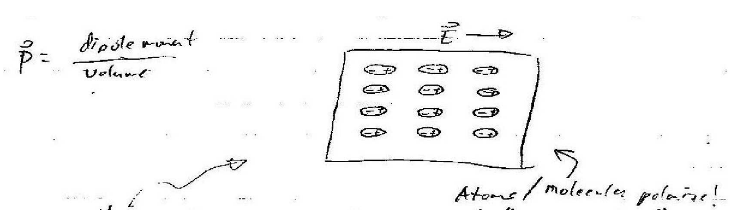
The great triumph of Maxwell's equations is the prediction of traveling E and M waves. (The behavior of a photon is closely linked to the classical $\vec{E}$ and $\vec{B}$ fields.) A wave solution for the electric field might look like:

$$\vec{E}\,(\vec{r},t) = \vec{E}_o cos\left(\vec{k}\cdot\vec{r} - \omega t\right) \tag{2}$$

We also have that the magnetic field must be perpendicular to the electric field for a wave solution, so $\vec{B}(\vec{r},t) = \vec{B}_o cos\left(\vec{k}\cdot\vec{r} - \omega t\right)$, with $\vec{B}_o$ perpendicular to $\vec{E}_o$. Maxwell's equations also force $\vec{E}$ and $\vec{B}$ to be perpendicular to the propagation vector $\vec{k}$, so we are given a natural orthogonal basis set for 3D, with $\vec{E}_o \times \vec{B}_o$ parallel to $\vec{k}$.

In cgs units, $|\vec{B}| = |\vec{E}|$, so let's ignore the $\vec{B}$-field entirely since if we know $\vec{E}$ then we know $\vec{B}$. Also in a material it's the $\vec{E}$-field that couples more strongly to electrons ($\vec{F} = q\vec{E}$), so the $\vec{E}$-field is more important.

So, we have traveling wave solutions. The phase velocity of any wave is given by $v_{ph} = \frac{\omega}{k} = \frac{c}{\sqrt{\varepsilon}}$, where $k = \frac{\omega}{c}\sqrt{\varepsilon}$. Perhaps more familiar is the index of refraction, $n = \sqrt{\varepsilon}$. For light, we have $v_{light} = \frac{c}{n}$, with $\sqrt{\varepsilon} = 1$ for vacuum. ε depends on material properties, and relates to the *polarizability* of a material: $\varepsilon = 1 + \frac{4\pi|\vec{p}|}{|\vec{E}|}$, where $\vec{p}$ = (dipole moment)/volume.



The more they polarize the bigger ε is and the slower light travels! Note: some materials might polarize more in one direction than another. The speed of light would thus be different for different polarizations of light.

Definition: The *polarization* of light is the direction that $\vec{E}$ points.

So what is a polarizer? This is a material that only allows light to pass with $\vec{E}$ polarized in a particular direction.

What about the energy of light? Classically a light wave fills a volume and has an energy density associated with it:

$$\rho = \frac{energy}{volume} = \frac{|\vec{E}|^2}{8\pi} \tag{3}$$

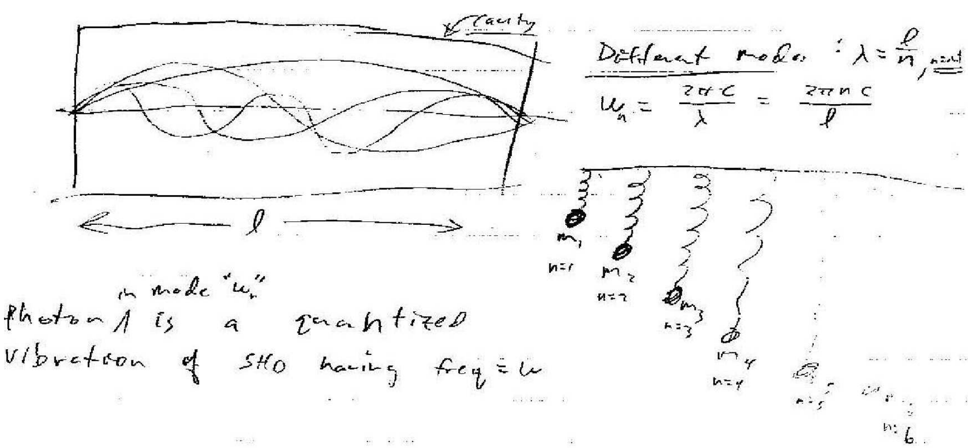
So the energy of a light wave in a volume of space (V) is:

$$U = \rho V = \frac{|\vec{E}|^2}{8\pi} V \tag{4}$$

Fact: Light actually travels in packets of energy called photons. Each photon has an $\vec{E}$-field and a frequency (ω) associated with it. In quantum mechanics, we know that the energy of a particle is proportional to its frequency, so we should have that $U_{photon} = \hbar\omega$. This is the *quanta* of energy associated with a the particle of light, the photon.

So why is light quantized? There are different ways to approach this. The full treatment is known as *quantum electro-dynamics*. We will not derive this, but if you are interested there is an excellent book by Richard Feynman on the subject entitled *QED*.

One way to think about the quantization of light is that if I have a light in a cavity (i.e. a box), the space inside the cavity can be thought of as a bunch of simple harmonic oscillators with different frequencies. Each frequency is a mode of the cavity, and just like waves on a string (or particle in a box!!), we have a discrete spectrum of allowable modes which fit in the cavity:

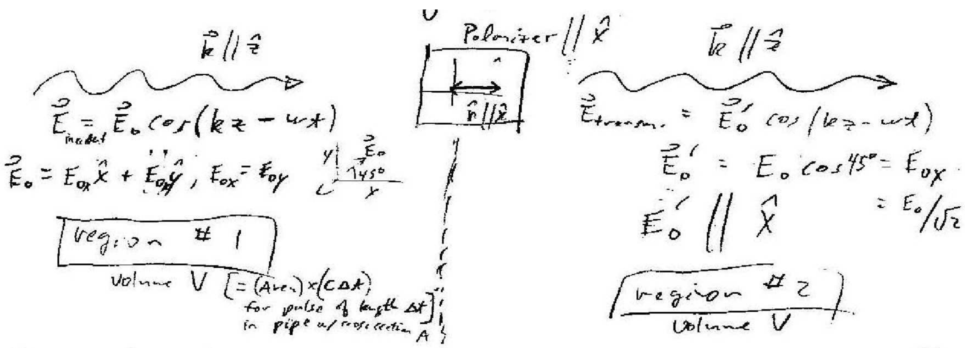


Consequence of Quantization:

1. The number of photons (N) in a light wave depends on magnitude of $\vec{E}$-field:

$$U = \frac{E_o^2}{8\pi} V = N\hbar\omega \qquad \Rightarrow \qquad N = \frac{E_o^2 V}{8\pi\hbar\omega} \tag{5}$$

2. Probabilistic behavior: Suppose we shine light on a polarizer. How do we interpret the behavior in terms of photons?



In region 1 we have N photons, all identical with $\vec{E}_o$ and ω. The number of photons in region 1 is given by $N = \frac{E_o^2 V}{8\pi\hbar\omega}$.

In region 2, we have N' photons, all identical with $\vec{E}_o'$ and ω. The number of photons in region 2 is given by $N' = \frac{E_o'^2 V}{8\pi\hbar\omega} = \frac{1}{8\pi}\frac{(E_o/\sqrt{2})^2 V}{\hbar\omega} = \frac{1}{2}\frac{E_o^2 V}{8\pi\hbar\omega}$. We then conclude that

$$\frac{N'}{N} = \frac{1}{2} \tag{6}$$

But this is strange since all the photons in region 1 are *identical*. Why do only half get through? Transmission must be probabilistic process, and we are therefore led to a probabilistic/QM interpretation. The probability of transmission for the preceding example is readily given by:

$$prob = \frac{E_{ox}^2}{E_{ox}^2 + E_{oy}^2} \tag{7}$$

Thus the components of the $\vec{E}$-field in different directions act as probability amplitudes, just like the probability for measuring $|0\rangle$ on a general state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ is:

$$prob = \frac{|\alpha|^2}{|\alpha|^2 + |\beta|^2} \tag{8}$$

So we can think of the components of an $\vec{E}$-field vector as a QM observable! The photon with electric field $\vec{E} = E_{ox}\hat{x} + E_{oy}\hat{y}$ can be thought of as living in a state $|\psi\rangle$ where:

$$|\psi\rangle = \begin{pmatrix} \psi_x \\ \psi_y \end{pmatrix} \tag{9}$$

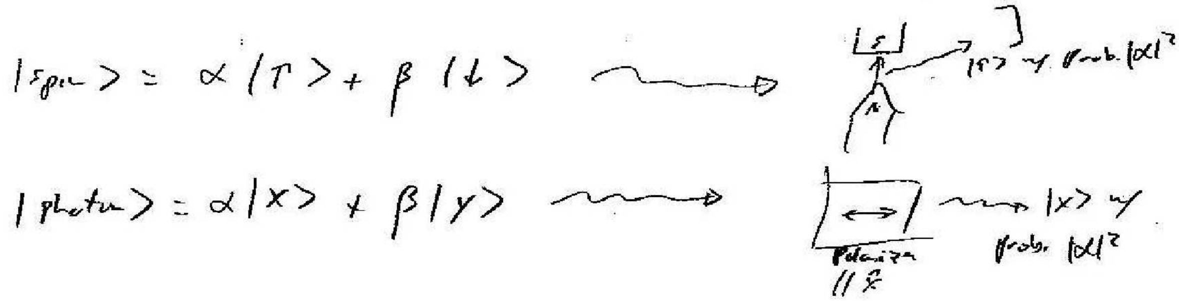
with ψ_x (ψ_y) being the x-polarized (y-polarized) component.

The $\hat{x}$ and $\hat{y}$ polarization vectors form a basis! Suppose we have $\vec{E}_o = E_{ox}\hat{x} + E_{oy}\hat{y}$. We can rewrite this state in familiar QM language:

$$\left|\psi\right\rangle = \begin{pmatrix} cos\theta \\ \sin\theta \end{pmatrix} = cos\theta\left|x\right\rangle + sin\theta\left|y\right\rangle \tag{10}$$

The polarization of the photon is our new qubit variable: $\left|0\right\rangle = \left|x\right\rangle$ and $\left|1\right\rangle = \left|y\right\rangle$. Nice! Now how do we measure and transform this qubit?

Here we again draw an analogy to spin, and the polarizer plays the same role a Stern-Gerlach device.



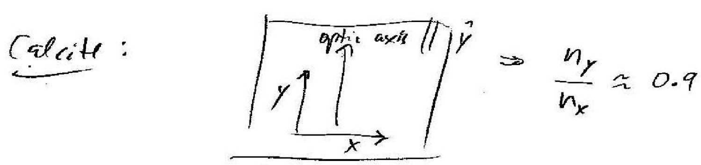
Consider the Bloch sphere with general quantum state vector $\left|\psi\right\rangle = cos\frac{\theta}{2}\left|0\right\rangle + e^{i\phi}sin\frac{\theta}{2}\left|1\right\rangle$. A polarizer at angle θ' w.r.t. $\hat{x}$ leads to:

$$\left|\psi\right\rangle_{photon} = cos\theta'\left|x\right\rangle + sin\theta'\left|y\right\rangle = cos\frac{\theta}{2}\left|0\right\rangle + e^{i\phi}sin\frac{\theta}{2}\left|1\right\rangle \tag{11}$$

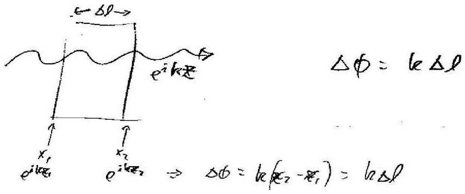
This equality leads us to identify $\theta = 2\theta'$ and $\phi = 0$ in this case.

For this to be analogous to spins, however, we must be able to vary ϕ. How do we do this? Is it even possible to get a relative phase terms with polarization?

The answer of course is yes, and the way this is done is to find a material where the index of refraction is different in $\hat{x}$ and $\hat{y}$ directions! This means that the velocity is different for $\hat{x}$ and $\hat{y}$ components. This is the case in *anisotropic* media with different polarizability in $\hat{x}$ and $\hat{y}$:



If this is the case, then light polarized along $\hat{y}$ will go $\approx 10\%$ faster than light polarized along the $\hat{x}$ direction. If the material has length Δl then we can calculate the phase difference for light passing through to be $\Delta\phi = k\Delta l$:



For light with $\vec{E}||\hat{y}$, we have $k_y = \frac{\omega}{c}n_y$. Similarly, for light with $\vec{E}||\hat{x}$, we have $k_x = \frac{\omega}{c}n_x$. Thus the phase change through the material for the y-component (x-component) is $\Delta\phi_y = \frac{\omega}{c}n_y\Delta l$ ($\Delta\phi_x = \frac{\omega}{c}n_x\Delta l$. So, if our input state is $\left|\psi\right\rangle_{in} = cos\frac{\theta}{2}\left|x\right\rangle + sin\frac{\theta}{2}\left|y\right\rangle$, then the output state is:

$$\left|\psi\right\rangle_{out} = cos\frac{\theta}{2}e^{i\frac{\omega}{c}n_x\Delta l}\left|x\right\rangle + sin\frac{\theta}{2}e^{i\frac{\omega}{c}n_y\Delta l}\left|y\right\rangle \tag{12}$$

Rewriting, we see:

$$\left|\psi\right\rangle_{out} = cos\frac{\theta}{2}\left|x\right\rangle + sin\frac{\theta}{2}e^{i\frac{\omega}{c}(n_y-n_x)\Delta l}\left|y\right\rangle \tag{13}$$

so ϕ on the the Bloch sphere is $\frac{\omega}{c}\Delta l(n_y - n_x)$. Therefore we see that this anisotropic medium has played the same role that the transverse $\vec{B}$-field did for spin.