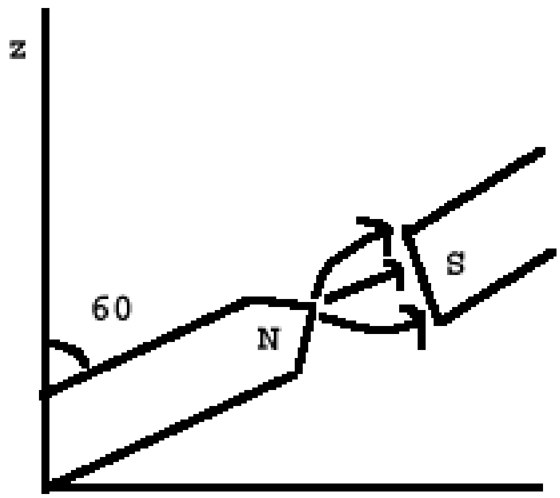
1. Using the "standard basis" (i.e. $|0\rangle$ and $|1\rangle$, eigenstates of $\hat{S}_z$), calculate the eigenvalues and eigenvectors associated with measuring the component of spin along an arbitrary direction $\hat{n}$ for a spin-$\frac{1}{2}$ system. ($S_{\hat{n}} = \hat{S}_x n_x + \hat{S}_y n_y + \hat{S}_z n_z$ as in class.) Show your work.

2. Suppose 10,000 electrons are prepared in the $|\Psi\rangle = |0\rangle$ spin state and then shot through a Stern-Gerlach device oriented with North $\rightarrow$ South magnet poles aligned 60° from the $+\hat{z}$-direction. About how many electrons go into each of the two resulting beams?



3. Consider a qubit comprised of a single spin-$\frac{1}{2}$ electron. How would you construct a NOT gate in the laboratory to act on this qubit? If your gate involves building a magnet, then please specify the exact orientation, amplitude, and time duration of any proposed applied magnetic fields.

4. An electron is shot with well-defined momentum at a 2-slit device. Does it get through to the detector? Consider the details below and explain your answer:

   In case you are worried about the different spatial trajectories the particles take, let's take a quick look at the full quantum state with the spatial dependence factored in:

   $|\psi\rangle_{electron} = |\psi_{spatial}\rangle \cdot |\psi_{spin}\rangle$

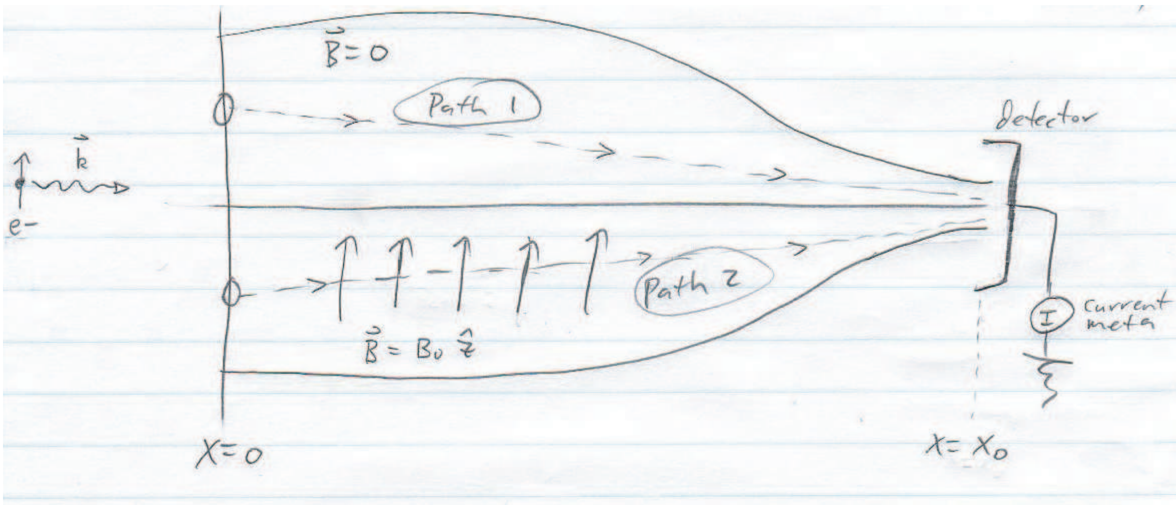
   $\psi_{electron}(x) = \langle x|\psi_{spatial}\rangle \cdot |\psi_{spin}\rangle = \psi_{spatial}(x)|\psi_{spin}\rangle = e^{ikx}|\psi_{spin}\rangle$

   We further specify that $\psi_{electron}(x) = |0\rangle$ and $\psi_{electron}(x = x_o) = \psi_{path1} + \psi_{path2}$, where:

   $\psi_{path1} = e^{ikx_o}|0\rangle$

   $\psi_{path2} = e^{ikx_o} e^{-i\frac{\hat{S}_z}{\hbar}\Delta\phi}|0\rangle$

Further, $\Delta\phi = \frac{eB_o}{m}\Delta t$ and $\Delta t = \frac{x_o}{v} = \frac{x_o}{\frac{\hbar k}{m}}$. Assume that we are clever enough to experimentally engineer $\Delta\phi = 2\pi$.



5. Consider the total spin of a system having 2 electrons: $\hat{S}_T = \hat{S}_1 + \hat{S}_2$. Show that the entangled state $|\psi\rangle_{total} = |0\rangle_1|1\rangle_2 - |1\rangle_1|0\rangle_2$ has a net spin of zero. In other words, show that it is an eigenstate of $\hat{S}_T^2$ with eigenvalue = 0.

   Hint: Use $\hat{S}_T^2 = \left(\hat{S}_1 + \hat{S}_2\right)^2 = \hat{S}_1^2 + \hat{S}_2^2 + 2\hat{S}_1 \cdot \hat{S}_2$, *and recall that* $\hat{S}_1 \cdot \hat{S}_2 = \hat{S}_{1x}\hat{S}_{2x} + \hat{S}_{1y}\hat{S}_{2y} + \hat{S}_{1z}\hat{S}_{2z}$.

6. Consider an atomic qubit system made from a single hydrogen atom. The qubit levels are the ground state and first excited state of hydrogen.

   $|0\rangle$ = ground state, E_o

   $|1\rangle$ = first excited state, E_1

   Suppose a laser at the resonant frequency $\omega = \frac{E_1 - E_o}{\hbar}$ is directed at the atom with an intensity such that $\langle 0|eE\hat{z}|1\rangle = 10^{-6}$ eV. How long should the laser be pointed at the atom to act as a Hadamard gate?