1. Consider an arbitrary qubit state $|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle$. If we think of this state as a vector on the Bloch sphere, then what is the action of a phase flip gate on the vector? (I.e. how does the vector get rotated on the Bloch sphere?)

2. Starting from the commutation relationship $[x, p] = i\hbar$ and the definition $\vec{L} = \vec{r} \times \vec{p}$, show that $[L_x, L_y] = i\hbar L_z$ and $[L^2, L_z] = 0$.

3. Find the eigenvectors of the $\hat{S}_x$ and $\hat{S}_y$ operators using the eigenvectors of $\hat{S}_z$ as a basis. Where do all of these eigenvectors sit on the Bloch sphere?

4. Consider two possible spin states:

$$\begin{aligned} |\Psi_1\rangle &= |0\rangle \\ |\Psi_2\rangle &= \sqrt{\frac{1}{4}}|0\rangle + i\sqrt{\frac{3}{4}}|1\rangle \end{aligned}$$

   a) What is the expectation value for S_z for these states? (Expectation value $= \langle S_z \rangle = \langle \Psi | S_z | Psi \rangle$)

   b) What is $\langle S_x \rangle$ for these states?

   c) What is $\langle S_y \rangle$ for these states?

5. Consider the spin state $|\Psi\rangle = \sqrt{\frac{1}{4}}|0\rangle + i\sqrt{\frac{3}{4}}|1\rangle$

   a) What is the probability that a measurement of the z-component of spin leads to $\hbar/2$?

   b) What is the probability that a measurement of the x-component of spin leads to $\hbar/2$?

   [Hint: use solution to problem #3 for this problem.]

6. Show that the Pauli Matrices are Hermitian.

7. Prove that $\hat{S}_-$ transforms the spin eigenstate $|s, m\rangle$ into the state $|s, m-1\rangle$.

8. Find A_+ and A_-, where

$$\begin{aligned} S_+|s,m\rangle &= A_+|s,m+1\rangle \\ S_-|s,m\rangle &= A_-|s,m-1\rangle \end{aligned}$$

   Show work!

   Hint: $|A_-|^2 = \langle s,m | S_+ S_- | s,m \rangle$

9. Calculate the 2×2 matrix that represents $\hat{S}_y$ for spin-$\frac{1}{2}$ systems.