1. You are given one of two quantum states of a single qubit: either $|\phi\rangle = |0\rangle$ or $|\psi\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle$. What measurement best distinguishes between these two states? If the state you are presented is either $|\phi\rangle$ or $|\psi\rangle$ with 50% probability each, what is the probability that your measurement correctly identifies the state? Can you generalize your result to distinguish between two arbitrary quantum states $|\phi\rangle$ and $|\psi\rangle$ on two qubits?

2. Alice and Bob share an arbitrarily long common string S. Alice is given as input a random bit a_{in} and Bob a random bit b_{in}. Without communicating with each other, Alice and Bob wish to output bits a_{out} and b_{out} respectively such that $a_{in} \cdot b_{in} = a_{out} + b_{out} \ (mod\ 2)$. Prove that any protocol that Alice and Bob follow has success probability at most $3/4$.

3. Prove that the Bell state $|\psi^-\rangle$ is rotationally invariant: i.e. $|\psi^-\rangle = \frac{1}{\sqrt{2}}(|vv^\perp\rangle - |v^\perp v\rangle)$.

4. Give a quantum circuit that outputs the four Bell states $|\psi^+\rangle$, $|\psi^-\rangle$, $|\phi^+\rangle$, $|\phi^-\rangle$ on input (respectively) the four basis states $|00\rangle, |01\rangle, |10\rangle, |11\rangle$ Use this to conclude that the Bell states form an othonormal basis for $\mathscr{C}^4$.