1. Consider a CNOT gate whose second (target) input is $|0\rangle - |1\rangle$. Describe the action of the CNOT gate on the first (control) qubit.

   Now show that if the CNOT gate is applied in the Hadamard basis - i.e. apply the Hadamard gate to the inputs and outputs of the CNOT gate - then the result is a CNOT gate with the control and target qubit swapped.

2. Suppose Alice and Bob use the teleportation protocol to teleport two qubits (i.e. assume they share two Bell pairs, and use a Bell pair to teleport each of the two qubits). What is the quantum state that Bob reconstructs in the case that the two qubits are entangled? For example, if they are themselves in one of the Bell basis states.

   Now, suppose that Alice has an arbitrary n qubit state, and Alice and Bob share n Bell states. What is the resulting state on Bob's side if they run the teleportation protocol on the n qubits. How many classical bits does Alice send to Bob? Can you reconcile the results with your intuitions about quantum states.

3. Suppose for some function f, there is an efficient classical circuit to compute $f(x)$ on input x, and an efficient classical circuit to compute x on input $f(x)$. Show that there is an efficient quantum circuit that computes $f(x)$ on input x (with a number of wires that start and end in the state $|0\rangle$, of course).

4. Suppose we ran m iterations of Grover's algorithm on some function f (with domain of size N, which has one marked element y) and the resulting superposition was exactly $|y\rangle$. What was the state after the $m - 1^{th}$ iteration? Note that you can describe the superposition by specifying two numbers α_y and α_x for $x \neq y$.

   Now, consider the case where $N/4$ elements are marked instead of just one. If we run one iteration of Grover's algorithm and measure, what is the probability that we see a marked element?

5. Suppose there are K solutions in the table of N items. Analyze the running time of Grover's algorithm for picking a uniformly random solution among the K possibilities.

   Now give a $O(\sqrt{N})$ quantum algorithm for finding the minimum element in a list of N numbers.