

1. Show that if you can factor N then you can efficiently and classically find the period of a periodic function mod N .
2. Show that phase estimation is complete for BQP.
3. Let v be the maximally entangled state on a d dimensional space H , i.e. $v = \frac{1}{\sqrt{d}} \sum_{i=1}^d |i\rangle |i\rangle$. Given two unitaries U, V acting on H , conclusively establish the relationship between these two quantities: $\langle (U \otimes V)v, v \rangle$ and $\text{tr}(UV)$, using tensor networks. (I.e. your thinking and proof should feature pictures!).
4. a. Recall that given a vector $v \in H \otimes H$, the Schmidt decomposition of v is of the following form:
 - $v = \sum_i d_i a_i \otimes b_i$, $a_i, b_i \in H$ and d_i non-negative numbers, such that
 - $\{a_i\}$ and $\{b_i\}$ are each orthonormal sets.

Draw the tensor network picture of the Schmidt decomposition and write down (as pictures) the conditions for orthonormality.

- b. Suppose you have a decomposition of the form $v = \sum_i f_i c_i \otimes d_i$, such that $\{c_i\}$ and $\{d_i\}$ are orthogonal sets but not normal, and f_i are complex numbers. Can you easily realize this as a Schmidt decomposition. . . how do you do that?