1. Alice, Bob and Charlie share a GHZ state, of 3 qubits:

$$\frac{1}{2}\left(|000\rangle - |011\rangle - |101\rangle - |110\rangle\right).$$

   (Warmup question: what is the state that results if after performing a Hadamard gate on each qubit?) They are given as input bits X_A, X_B, X_C respectively, which satisfy the condition $X_A \oplus X_B \oplus X_C = 0$. They wish to output a, b, c s.t. $X_A \vee X_B \vee X_C = a \oplus b \oplus c$. What is the maximum probability with which you can achieve this classically? Give a quantum protocol to achieve this with certainty. (Hint: what happens if each player performs a Hadamard gate or not on their qubit depending upon their input bit).

2. Generalize Grover's search algorithm to the case where there are K solutions in a list of N items. What is the running time as a function of N and K.

3. Given a $2-1$ function $f : [N] \to [N]$, we wish to find a collision. i.e. x, y such that $f(x) = f(y)$. Show that the quantum query complexity for this collision problem is $O(N^{1/3} logN)$.

   Hint: Pick a random sample of $O(N^{1/3})$ elements and use Grover's algorithm to find a collision of an x in this sample and a y not in this sample.

4. Give a linear time quantum algorithm for the following problem: given n numbers $x_1, \ldots, x_n$, decide whether they are all distinct.