1. Alice, Bob and Charlie share a GHZ state, of 3 qubits:

\[ \frac{1}{2} (|000\rangle - |011\rangle - |101\rangle - |110\rangle) . \]

(Warmup question: what is the state that results if after performing a Hadamard gate on each qubit?) They are given as input bits \( X_A, X_B, X_C \) respectively, which satisfy the condition \( X_A \oplus X_B \oplus X_C = 0 \). They wish to output \( a, b, c \) s.t. \( X_A \lor X_B \lor X_C = a \oplus b \oplus c \).

What is the maximum probability with which you can achieve this classically? Give a quantum protocol to achieve this with certainty. (Hint: what happens if each player performs a Hadamard gate or not on their qubit depending upon their input bit).

2. Generalize Grover’s search algorithm to the case where there are \( K \) solutions in a list of \( N \) items. What is the running time as a function of \( N \) and \( K \).

3. Given a \( 2^d - 1 \) function \( f : [N] \rightarrow [N] \), we wish to find a collision. i.e. \( x, y \) such that \( f(x) = f(y) \).

Show that the quantum query complexity for this collision problem is \( O(N^{1/3} \log N) \).

Hint: Pick a random sample of \( O(N^{1/3}) \) elements and use Grover’s algorithm to find a collision of an \( x \) in this sample and a \( y \) not in this sample.

4. Give a linear time quantum algorithm for the following problem: given \( n \) numbers \( x_1, \ldots, x_n \), decide whether they are all distinct.