1. Suppose you are given quantum circuits C_i for computing the Fourier transform mod n_i for $i = 1$ *to* k where the n_i are pairwise relatively prime. Give a quantum circuit for computing the Fourier transform mod N where $N = n_1 \cdot n_2 \cdots n_k$. Bound the size of the resulting ciruit in terms of N and the sizes of the given circuits C_i.

2. Let $a|q$ and $b|q$. What is the Fourier transform mod q of the uniform superposition on all $0 \leq x < q$ such that $a|x$ or $b|x$.

3. Recall the experiment demonstrating Bell inequality violations: Alice and Bob share entanglement and on input bits a and b output bits x_A and x_B such that $a \cdot b = x_A + x_B$ $(mod\ 2)$ with probability greater than 3/4.

   - Show that if Alice and Bob could output x_A and x_B satisfying the above condition with certainty, then Bob could reconstruct a without any communication from Alice, thus achieving superluminal communication.

     Hint: suppose Bob sets his input b to be a superposition of $|0\rangle$ and $|1\rangle$.
   - Now consider the generalization where the inputs a and b are numbers mod N, and Alice and Bob output x_A and x_B mod N such that $a \cdot b = x_A + x_B$ $(mod\ N)$. Prove that if Alice and Bob can achieve this with certainty then Bob can reconstruct a without any communication from Alice.

4. Suppose there are K solutions in the table of N items. Analyze the running time of Grover's algorithm for picking a uniformly random solution among the K possibilities.

   Now give a quantum algorithm for finding the minimum element in a list of N numbers.

5. Suppose you are given a 2^k-to-1 function $f : \{0,1\}^n \rightarrow \{0,1\}^n$ such that there exist n-bit strings $a_1, \ldots, a_k$, such that for all $x \in \{0,1\}^n$ and for $1 \leq i \leq k$, $f(x + a_i) = f(x)$. What information about the a_i's can we hope to reconstruct from f? Cast this as an instance of the hidden subgroup problem. What is the underlying group G. What is the hidden subgroup H? Work out the details of the algorithm (including the classical reconstruction).