1. Generalize Grover's search algorithm to the case where there are K solutions in a list of N items. What is the running time as a function of N and K.

2. Suppose you wish to estimate the number K of solutions to with in a multiplicative factor of ϵ. Give a quantum algorithm for this problem.

3. Give a quantum algorithm for finding the minimum element in a list of N numbers.

4. Given a $2-1$ function $f : [N] \rightarrow [N]$, we wish to find a collision. i.e. x, y such that $f(x) = f(y)$. Show that the quantum query complexity for this collision problem is $O(N^{1/3} log N)$.

   Hint: Pick a random sample of $O(N^{1/3})$ elements and use Grover's algorithm to find a collision of an x in this sample and a y not in this sample.