1. Consider the geometric series $1/N \sum_{j=1}^{N} \beta^j$, where $\beta \in \mathcal{C}$ has unit magnitude. If β^N wraps around k times (i.e. N times the phase of β is at greater than $2\pi k$), and if the phase of β lies between 0 and π, then show that the sum is at most $1/k$.

2. Consider the quantum circuit for computing the Fourier transform mod2^n presented in class. Now, assume that you only want to compute the Fourier transform approximately to with in ϵ (i.e. in operator norm). How much more efficient can you make the circuit.

3. Let $|v\rangle = \sum_x \alpha_x |x\rangle$ be an n qubit state such that for all x, α_x is real and non-negative. Suppose you have a procedure that on input $0 \leq j, k < 2^n$, outputs $\sum_{x=j}^{k} |alpha_x|^2$. Give a polynomial size quantum circuit that on input $|v\rangle$ outputs $|0^n\rangle$.

4. Prove that in the phase estimation procedure, that if the eigenvalue is $\lambda = e^{2\pi i\theta}$, where $\frac{a-1}{2^m} \leq \theta \leq \frac{a}{2^m}$, then the phase estimation procedure outputs a or $a-1$ with high probability.

5. Let U be the unitary transformation implemented by a quantum circuit, and let $|\phi_j\rangle$ be its eigenvectors with eigenvalues $\lambda_j = e^{2\pi i\theta_j}$. Define $\sqrt{U}$ to be the unitary transformation with the same eigenvectors $|\phi_j\rangle$, but whose eigenvalues are $\sqrt{\lambda_j}$. Show that you can use phase estimation to implement an approximation to $\sqrt{U}$, which has eigenvectors $|\phi_j\rangle$, with eigenvalues $\mu_j = e^{2\pi i\beta_j}$, where $(1-1/n)\theta_j/2 \leq mu_j \leq (1+1/n)\theta_j/2$.