1. Prove that if U and V are unitary then so is $U \otimes V$.

2. Generalize quantum teleportation to qutrits — $|\psi\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle + \alpha_2|2\rangle$. Hint: use a fourier transform gate F_3 in place of the Hadamard gate — F_3 maps the basis states of a qutrit as follows: $|j\rangle \rightarrow \sum_k \omega^{jk}|k\rangle$, where $\omega = e^{2\pi i/3}$ is the principal third root of unity. Also use the sum modulo 3 gate in place of the controlled not: this gate maps $|j\rangle \otimes |k\rangle$ to $|j\rangle \otimes |j + k \bmod 3\rangle$.

3. Let $\mathcal{D}(\psi)$ denote the probability distribution that results from measuring the superposition $|\psi\rangle$. Prove that if $|||psi\rangle - |\phi\rangle|| \leq \epsilon$ then $|\mathcal{D}(\psi) - \mathcal{D}(\phi)| \leq \triangle\epsilon$.

4. Let $f : \{0,1\}^n \rightarrow \{0,1\}$. Let U_f be the unitary transformation that on input $|x\rangle|b\rangle$ (where $x \in \{0,1\}^n$ and $b \in \{0,1\}$), outputs $|x\rangle|b \oplus f(x)\rangle$. Show that if the answer bit b is initialized to $|0\rangle - |1\rangle$ then then on input $|x\rangle(|0\rangle - |1\rangle)$, the output is $(-1)^{f(x)}|x\rangle(|0\rangle - |1\rangle)$. i.e. the answer bit remains unchanged, and effectively the transformation that is achieved is: $|x\rangle \rightarrow (-1)^{f(x)}|x\rangle$.