1. Prove the no cloning theorem: there is no procedure to clone an unknown quantum state. i.e. given a state $|\psi\rangle$ produce a state $|\psi\rangle \otimes |\psi\rangle$.

   Reconcile this theorem with a) the copying action of the CNOT gate. b) quantum teleportation.

2. Consider a CNOT gate whose second input is $|0\rangle - |1\rangle$. Describe the action of the CNOT gate on the first qubit.

   Now show that if the CNOT gate is applied in the Hadamard basis - i.e. apply the Hadamard gate to the inputs and outputs of the CNOT gate - then the result is a CNOT gate with the control and target qubit swapped.

3. Prove that we can assume without loss of generality that all amplitudes in a quantum computation are real numbers, by showing that any quantum circuit with 2 qubit gates can be simulated with an equivalent quantum circuit with 3 qubit gates (but the number of gates is the same), such that all amplitudes in the simulating circuit are real.

4. Consider a bipartite quantum state $|\psi\rangle \in \mathcal{H}_{\mathcal{A}} \otimes \mathcal{H}_{\mathcal{B}}$. Show that if A performs an arbitrary unitary operation on her part of $|\psi\rangle$ and then B measures his qubits in the standard basis then the result of B's measurement is independent of A's actions.

   Now show that there is no measurement that B can perform to tell which unitary operation A performed on her qubits.

5. Write the $4 \times 4$ matrix of the unitary operation on two qubits resulting from performing a Hadamard transform on the first qubit and a phase flip on the second qubit.