1. Alice and Bob share an arbitrarily long common random string. Alice is given as input a random bit x_A and Bob a random bit x_B. Without communicating with each other, Alice and Bob wish to output bits a and b respectively such that $x_A \wedge x_B = a \oplus b$. Prove that any protocol that Alice and Bob follow has success probability at most 3/4.

2. Alice, Bob and Charlie share a GHZ state, of 3 qubits:

$$\frac{1}{2}\left(|000\rangle - |011\rangle - |101\rangle - |110\rangle\right).$$

   They are given as input bits X_A, X_B, X_C respectively, which satisfy the condition $X_A \oplus X_B \oplus X_C = 0$. They wish to output a, b, c s.t. $X_A \vee X_B \vee X_C = a \oplus b \oplus c$. Give a quantum protocol to achieve this with certainty. What is the maximum probability with which you can achieve this classically?

3. You are given one of two quantum states of a single qubit: either $|\phi\rangle = |0\rangle$ or $|\psi\rangle = cos\theta|0\rangle + sin\theta|1\rangle$. What measurement best distinguishes between these two states? If the state you are presented is either $|\phi\rangle$ or $|\psi\rangle$ with 50% probability each, what is the probability that your measurement correctly identifies the state?