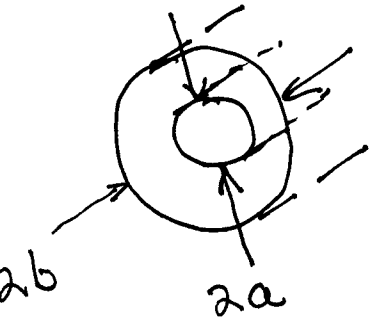
Transmission Line Equations are thus

$$\frac{\partial V}{\partial x} = -L\frac{\partial I}{\partial t}$$
$$\frac{\partial I}{\partial x} = -C\frac{\partial V}{\partial t}$$

} Eq^{ns} (2.18) with $R' = 0$, $G' = 0$, $L = L'$, $C = C'$

For the coaxial cable in particular



$$L = \frac{\mu}{2\pi}\ln b/a \quad [H/m]$$

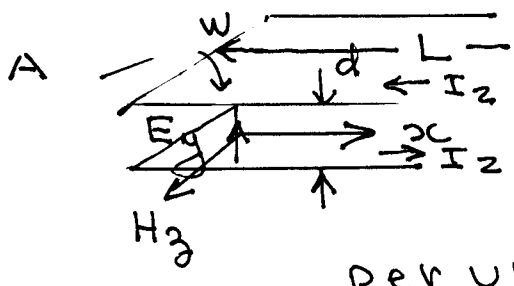
$$C = \epsilon\frac{2\pi}{\ln b/a} \quad [F/m]$$

so $LC = \mu\epsilon$ (Fundamental)

Other types of transmission lines are shown in Fig 2-4

In table 2-1 look at L', C' for coaxial and parallel plate

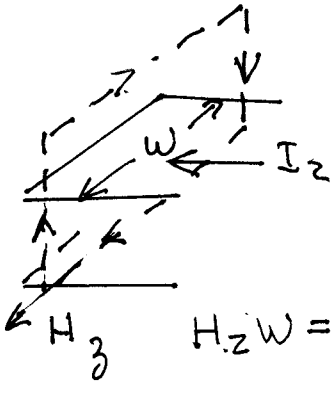
Parallel Plate - Like a capacitor except that the voltage varies with x



$$C = \frac{\epsilon A}{d} \quad [F] \quad \text{for capacitor}$$

per unit length

$$C = \frac{\epsilon A/L}{d} = \frac{\epsilon w}{d} \quad [F/m]$$



Flux per unit length

$$= Bd = \frac{\mu_0 d H_z}{\text{current}} = \frac{\mu_0 d\, I_x/w}{I_x}$$

$$= \frac{\mu_0 d}{w}$$

$H_z w = I_z$

<u>Generally</u> use lower case v and i for voltage and current with general x and t dependence, capitals for sin & cos

<u>General Properties</u>, of The Transmission Line Equations (Use z for the propagation direction)

$$\frac{\partial v}{\partial z} = -L\frac{\partial i}{\partial t} \quad (A) \quad ; \quad \frac{\partial i}{\partial z} = -C\frac{\partial v}{\partial t} \quad (B)$$

① v and i satisfy the wave equations with general solutions

$$v(z,t) = v^+(t - z/v_{ph}) + v^-(t + z/v_{ph})$$

$$i(z,t) = i^+(t - z/v) + i^-(t + z/v_{ph})$$

$$v_{ph} = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\mu\epsilon}}$$ (see previous page coaxial line. This is general for <u>T.E.M.</u> waves

② For an infinite line v^+ and v^- are independent. For finite lines the <u>end conditions</u> are important and relate the two (boundary conditions)

③ v^+ is related to i^+ through A or B.

$$v^+ = v^+(t - z/v_{ph}) \; ; \; i^+ = i^+(t - z/v_{ph})$$

For instance use (A) above

$$\frac{\partial v^+}{\partial z} = \frac{dv^+}{d(t - z/v_{ph})}\left(-\frac{1}{v_{ph}}\right) \; ; \; \frac{\partial i^+}{\partial t} = \frac{di^+}{d(t - z/v_{ph})}$$

Thus $$\frac{dv^+}{d(t - z/v_{ph})}\left(-\frac{1}{v_{ph}}\right) = \frac{di^+}{d(t - z/v_{ph})}(-L$$

Integrate w.r.t. $t - z/v_{ph}$

$$v^+ \left(-1/v_{ph}\right) = i^+ (-L) + (\text{Const})$$

Const = 0 since there is no v^+ without an i^+

<u>Consequently</u>

$$v^+/i^+ = L v_{ph} = L \frac{1}{\sqrt{LC}} = \sqrt{\frac{L}{C}} = +Z_0$$

Z_0 is called the characteristic impedance

Similarly show that

$$v^-/i^- = -L v_{ph} = -Z_0$$

Thus $v^+/i^+ = +(\text{characteristic impedance}) = Z_0$

and $v^-/i^- = -(\text{charaterstic impedance}) = -Z_0$

Recall : Analogy with waves on a rope

$y \leftrightarrow$ voltage $\qquad \frac{dy}{dx} \leftrightarrow$ current

End conditions

fixed end on rope $\leftrightarrow$ short circuit ($y=0$)

zero slope on end of rope $\leftrightarrow$ open circuit

cases of most interest are intermediary

It was stated that v and i satisfy the wave equation
To establish this consider

$$\frac{\partial v}{\partial z} = -L\frac{\partial i}{\partial t} \quad (A)$$

$$\frac{\partial i}{\partial z} = -C\frac{\partial v}{\partial t} \quad (B)$$

Take $\frac{\partial}{\partial z}(A) = \frac{\partial^2 v}{\partial z^2} = -L\frac{\partial^2 i}{\partial z\,\partial t} = -L\frac{\partial^2 i}{\partial t\,\partial z}$

$$= -L\frac{\partial}{\partial t}\left(\frac{\partial i}{\partial z}\right)$$ substitute from B

$$= -L\frac{\partial}{\partial t}\left(-C\frac{\partial v}{\partial t}\right)$$

$$= +LC\frac{\partial^2 v}{\partial t^2}$$

Thus $\frac{\partial^2 v}{\partial z^2} - LC\frac{\partial^2 v}{\partial t^2} = 0$

Note: This can be written as

$$\left(\frac{\partial}{\partial z} - \sqrt{LC}\frac{\partial}{\partial t}\right)\left(\frac{\partial}{\partial z} + \sqrt{LC}\frac{\partial}{\partial t}\right)v(t,z) = 0$$

Solution $v(t,z) = v^+(t+\sqrt{LC}\,z) + v^-(t-\sqrt{LC}\,z)$

since $\left(\frac{\partial}{\partial z} + \sqrt{LC}\frac{\partial}{\partial t}\right)v^+(t+\sqrt{LC}\,z) = 0$

and $\left(\frac{\partial}{\partial z} - \sqrt{LC}\frac{\partial}{\partial t}\right)v^-(t-\sqrt{LC}\,z) = 0$