Transmission Line Equations are thus

\[
\begin{align*}
\frac{dV}{dx} &= -L \frac{dI}{dt} \quad & \text{Eq. (2.18)} \\
\frac{dI}{dx} &= -C \frac{dV}{dt}
\end{align*}
\]

with \( R' = 0 \), \( G' = 0 \)

\( L = L', \quad C = C' \)

For the coaxial cable in particular

\[
L = \frac{√}{2\pi} \ln \frac{b}{a} \quad [\text{H/m}]
\]

\[
C = \frac{\varepsilon}{2\pi} \left( \frac{\ln \frac{b}{a}}{\ln \frac{b}{a}} \right) \quad [\text{F/m}] 
\]

So \( LC = \mu E \) (Fundamental)

Other types of transmission lines are shown in Fig 2-4

In table 2-1 look at \( L', c' \) for coaxial and parallel plate

**Parallel Plate** - Like a capacitor except that the voltage varies

\[
C = \varepsilon A \quad \text{for capacitor per unit length} \quad \frac{d}{d}
\]

\[
C = \varepsilon \frac{A}{L} = \varepsilon \frac{w}{d} \quad [\text{F/m}]
\]

Flux per unit length

\[
Bd = \mu d \frac{H_z}{w} \quad \frac{\text{current}}{I_x} = \frac{\mu d}{w}
\]
Generally use lower case \( v \) and \( i \) for voltage and current with general \( x \) and \( t \) dependence, capital \( V \) and \( I \) for sin \& cos dependence.

**General Properties of The Transmission Line Equations** (Use \( z \) for the propagation direction)

\[
\begin{align*}
\frac{dV}{dz} &= -L \frac{di}{dt} \quad \frac{di}{dt} &= -C \frac{dV}{dz}
\end{align*}
\]

1) \( V \) and \( i \) satisfy the wave equations with general solutions:

\[
\begin{align*}
V(x,t) &= V^+(t - \frac{x}{v_{ph}}) + V^-(t + \frac{x}{v_{ph}}) \\
I(x,t) &= I^+(t - \frac{x}{v_{ph}}) + I^-(t + \frac{x}{v_{ph}})
\end{align*}
\]

\[
\frac{V^+}{\sqrt{LC}} = \frac{I^+}{\sqrt{\mu e}} \text{ (see previous page coaxial line. This is general for T.E.M. waves)}
\]

2) For an infinite line \( V^+ \) and \( V^- \) are independent. For finite lines, the end conditions are important and relate the two (boundary conditions).

3) \( V^+ \) is related to \( I^+ \) through \( A \) or \( B \).

\[
V^+ = V^+(t - \frac{x}{v_{ph}}) \quad I^+ = I^+(t - \frac{x}{v_{ph}})
\]

For instance, use \( (A) \) above:

\[
\begin{align*}
\frac{dV^+}{dz} &= \frac{dV^+}{d(t-\frac{x}{v_{ph}})} (-\frac{1}{\lambda_{ph}}) \\
\frac{di^+}{dt} &= \frac{dI^+}{d(t-\frac{x}{v_{ph}})} (-\frac{1}{\lambda_{ph}})
\end{align*}
\]

Thus

\[
\frac{dV^+}{d(t-\frac{x}{v_{ph}})} (-\frac{1}{\lambda_{ph}}) = \frac{di^+}{d(t-\frac{x}{v_{ph}})} (-L)
\]
Integrate w.r.t. $t - z_j \sqrt{\nu \phi}$

$N^+ (-\frac{1}{\sqrt{\nu \phi}}) = N^+ (-l) + (\text{Const})$

$\text{Const} = 0$ since there is no $N^+$ without an $\omega^+$

Consequently

$\frac{N^+}{\omega^+} = \frac{\ln \phi + \frac{l}{\sqrt{\nu \phi}}}{\sqrt{\nu \phi}} + Z_0$

$Z_0$ is called the characteristic impedance

Similarly show that

$\frac{N^-}{\omega^-} = -l \ln \phi = -Z_0$

Thus $\frac{N^+}{\omega^+} = + \text{(characteristic impedance)}$

and $\frac{N^-}{\omega^-} = - \text{(characteristic impedance)}$

Recall: Analogy with waves on a rope

$y \leftrightarrow$ voltage

$\frac{dy}{dt} \leftrightarrow$ current

End conditions

fixed end on rope $\leftrightarrow$ short circuit $(\phi = 0)$

zero slope on end of rope $\leftrightarrow$ open circuit

cases of most interest are intermediary
It was stated that $\nu$ and $i$ satisfy the wave equation. To establish this consider

$$\frac{\partial \nu}{\partial z} = -L \frac{\partial i}{\partial t} \quad \text{(A)}$$
$$\frac{\partial i}{\partial z} = -C \frac{\partial \nu}{\partial t} \quad \text{(B)}$$

Take $\frac{\partial}{\partial z} \text{(A)} = \frac{\partial^2 \nu}{\partial z^2} = -L \frac{\partial^2 i}{\partial z \partial t} = -L \frac{\partial^2 i}{\partial t \partial z} \biggl|_{\text{substitute from (B)}}$

$$= -L \frac{\partial}{\partial t} \left( -C \frac{\partial \nu}{\partial t} \right)$$

$$= -L \frac{\partial}{\partial t} \left( -C \frac{\partial \nu}{\partial t} \right)$$

$$= +LC \frac{\partial^2 \nu}{\partial t^2}$$

Thus $\frac{\partial^2 \nu}{\partial z^2} - LC \frac{\partial^2 \nu}{\partial t^2} = 0$

Note: This can be written as

$$\left( \frac{\partial}{\partial z} - \sqrt{LC} \frac{\partial}{\partial t} \right) \left( \frac{\partial}{\partial z} + \sqrt{LC} \frac{\partial}{\partial t} \right) \nu(t, z) = 0$$

Solution $\nu^+(t, z) = \nu^+(t + \sqrt{LC} z) + \nu^-(t - \sqrt{LC} z)$

since $\left( \frac{\partial}{\partial z} + \sqrt{LC} \frac{\partial}{\partial t} \right) \nu^+(t + \sqrt{LC} z) = 0$

and $\left( \frac{\partial}{\partial z} - \sqrt{LC} \frac{\partial}{\partial t} \right) \nu^-(t - \sqrt{LC} z) = 0$