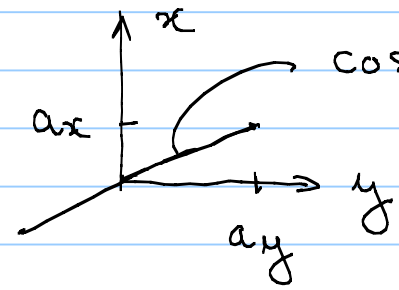
Polarization of Fields.

Note Title 4/19/2010

- Direction of Electric Field

linear

$$\underline{E}(0,t) = (\hat{x}\, a_x + \hat{y}\, a_y)\cos\omega t$$

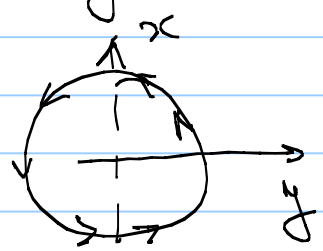


cosine oscillation along line

Circular.

$$\underline{E}(0,t) = (\hat{x}\, a_x \cos\omega t + \hat{y}\, a_y \sin\omega t)$$

$a_y = a_x$



+ helicity

In general elliptical page 300

Plane waves in lossy media

$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$$

$$\bar{E} = \frac{\tilde{E}}{2} e^{j\omega t - jkz} + c.c.$$

$$= \mathrm{Re}\left(\tilde{E} e^{j\omega t - jkz}\right)$$

$\therefore$ In phasor form

$$\nabla \times \bar{E} = -j\bar{k} \times \frac{\tilde{E}}{2} e^{j\omega t - j\bar{k}\cdot\bar{r}} + c.c.$$

take only $e^{j\omega t}$

$$-j\bar{k} \times \tilde{E} = -i\omega\mu\tilde{H} \quad (1)$$

$$\nabla \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}$$

$$\bar{J} = \sigma \bar{E}$$

gives

$$-j\bar{k} \times \tilde{H} = \bar{J} + j\omega\epsilon\tilde{E} \quad (2)$$

$$\therefore +j\bar{k} \times (j\bar{k} \times \tilde{E}) = -i\omega\mu(j\bar{k} \times \tilde{H})$$

Sub from (2)

$$+j\bar{k} \times (j\bar{k} \times \tilde{E}) = -j\omega\mu(\sigma + i\omega\epsilon)\tilde{E}$$

$$-\bar{k}(\underbrace{\bar{k}\cdot\bar{E}}_{0 \text{ from Gauss}}) + k^2\tilde{E} = -j\omega\sigma\tilde{E} + \omega^2\mu\epsilon\tilde{E}$$

$$k^2 = \omega^2 \mu \left(\overset{\epsilon'}{\epsilon} - j \overset{\epsilon''}{\left(\frac{\sigma}{\omega}\right)} \right)$$

$$k = \beta - j\alpha \quad - \text{complex} \quad e^{-jkz} \rightarrow e^{-\alpha z} \text{ decay}$$

$$-\alpha^2 + \beta^2 - 2j\alpha\beta = \omega^2 \mu \epsilon' - j\omega^2 \mu \epsilon''$$

equal

equal

Eq$^{\underline{ns}}$ (7.66) of text.

$$\eta_c = \sqrt{\frac{\mu}{\epsilon_c}} = \sqrt{\frac{\mu}{\epsilon' - j\epsilon''}} \rightarrow \text{intrinsic impedance}$$

$$\delta_s = \frac{1}{\alpha} = \text{"skin' depth}$$

if ϵ'' is large

$$\alpha = \omega \left\{ \frac{\mu}{2} \epsilon'' \right\}^{1/2}$$

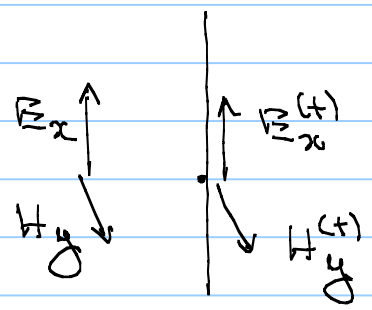
$$= \omega \left\{ \frac{\mu}{2} \frac{\sigma}{\omega} \right\}^{1/2} = \left\{ \mu \frac{\omega}{2} \sigma \right\}^{1/2}$$

$$= \left\{ \mu \pi f \sigma \right\}^{1/2}$$

Above $\alpha \simeq \beta$ (ϵ' small)

$$\eta_c = \sqrt{\mu j \frac{\omega}{\sigma}} = \left(\frac{1+j}{\sqrt{2}} \right) \sqrt{\frac{\mu 2\pi f}{\sigma}}$$

Surface Impedance



$$E_x^{(t)}(z) = E_x^{(t)}(0)\, e^{-j\frac{z}{\delta} - \frac{z}{\delta}} \times e^{j\omega t}$$

$$\therefore \quad j = \sigma E$$

$$\int j \, dz = \frac{\sigma \overset{\frac{V}{m}}{E_x^{(t)}(0)}}{\frac{1}{\delta}(1+j)} = \overset{\frac{\text{Amps}}{m}}{I_s}$$

$$\left(\frac{\delta\sigma}{1+j}\right) = \text{surface impedance}$$

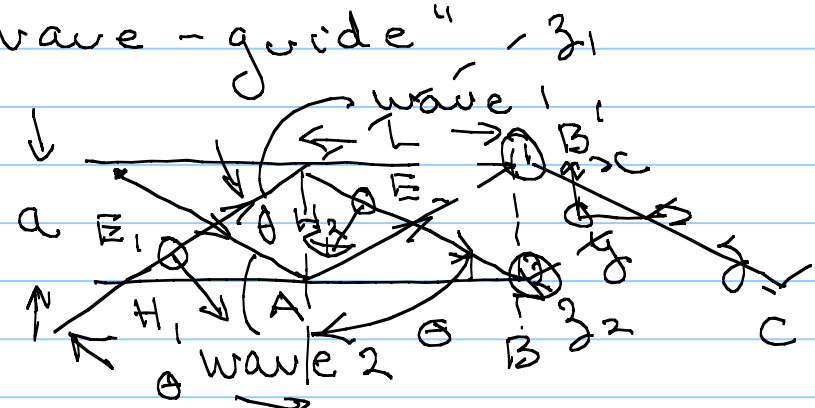
total current is $I_s w = I$

total voltage is $E_x^{(t)} l = V$

$$\therefore \quad \frac{V}{Z_s l} = I/w$$

$$Z = \frac{Z_s l}{w}$$

A "wave-guide"



characteristic of a guided wave

$$e^{j\omega t - jkz}$$

$k \neq \omega\sqrt{\mu\epsilon}$ but is the same for all x (plane phase front)

What condition needs to apply

wave 1 $\quad \tilde{E}_1 e^{j\omega t}\,\hat{y}\, e^{-jk\sin\theta z - jk\cos\theta x}$

$\omega\sqrt{\mu\epsilon}$

wave 2 $\quad \tilde{E}_2 e^{j\omega t}\,\hat{y}\, e^{-jk\sin\theta z + jk\cos\theta x}$

Thus in going from A to B wave 1 has phase change $-kL\sin\theta - k\cos\theta\, a$ and wave 2 has phase change $-kL\sin\theta - k\cos\theta\, a$

If we follow wave 1 just after reflection from A to B (with a reflection at B') to C and a final reflection at C, then the total phase change is $2k\sin\theta(2L) + 2k\cos\theta\, a + \Phi_1 + \Phi_2$ where Φ_1 and

Φ_2 are the phase changes upon reflection at B and C respectively.

For a mode to propagate this must reduce to only the phase change due to the z-component of $\underline{k}$ that is $2k\sin\theta(2L)$. Thus the mode condition is

$$2k\sin\theta\, a + \Phi_1 + \Phi_2 = n(2\pi)$$

where n is an integer.

For a metal $\Phi_1 = \Phi_2 = \pi$ so $\Phi_1 + \Phi_2$ does not enter. Thus in this case

$$ka\sin\theta = n\pi/a$$

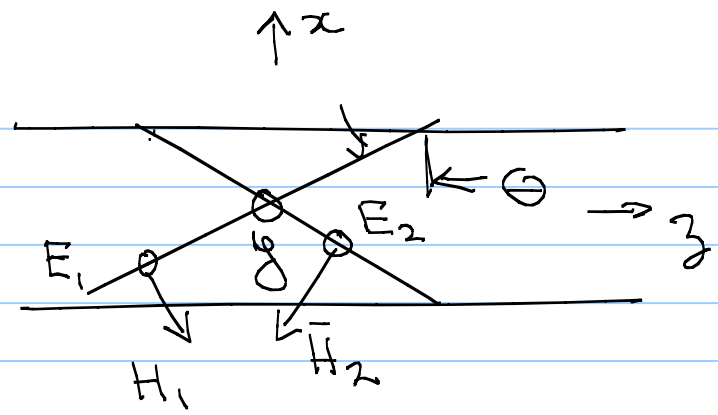
Thus θ is quantized

Total y field ($\tilde{E}_2 = -\tilde{E}_1$ by B.C. (Tangent $E = 0$ at metal boundary.

$$\tilde{E}_y = \tilde{E}_1 e^{j(\omega t - k_z z)}\, 2j \sin(k\cos\theta\, y)$$

$$= \tilde{E}_1 e\, 2j \sin\left(\frac{n\pi}{a}\right)$$

Now we can get $\tilde{H}_x$ and $\tilde{H}_z$ easily as well!



$$H_1 = E_1 / \sqrt{\mu/\epsilon} \qquad H_2 = E_2 / \sqrt{\mu/\epsilon} \qquad E_2 = -E_1$$

$$\tilde{H}_1 = H_1 e^{-jk_z z + jk_x x}$$

$$\tilde{H}_2 = H_2 e^{-jk_z z - jk_x x}$$

$$\tilde{H}_z = \frac{E_1}{\sqrt{\mu/\epsilon}} e^{-jk_z z} \cos\theta \, e^{-jk_x x} - \frac{E_2}{\sqrt{\mu/\epsilon}} \cos\theta \, e^{+jk_x x}$$

$$= +\frac{E_1}{\sqrt{\mu/\epsilon}} \cos\theta \; 2\cos(k_x x)$$

$$\tilde{H}_x = \frac{E_1}{\sqrt{\mu/\epsilon}} e^{-jk_z z} \sin\theta \, e^{-jk_x x} + \frac{E_2}{\sqrt{\mu/\epsilon}} \sin\theta \, e^{+jk_x x}$$

$$= -\frac{E_1}{\sqrt{\mu/\epsilon}} 2j\sin\theta (\sin k_x x)$$

This is a TE_n mode, $\tilde{H}_x = 0$ at both boundaries and $k_x a = n\pi$