Polarization of Fields.

- Direction of Electric Field

Linear

\[ E(x,t) = (\hat{x} a_x + \hat{y} a_y) \cos \omega t \]

Circular

\[ E(x,t) = (\hat{x} a_x \cos \omega t + \hat{y} a_y \sin \omega t) \]

In general elliptical page 300
Plane waves in lossy media

\[ \nabla \times \vec{E} = -\sigma \vec{E} \]

\[ \vec{E} = \frac{\hat{\sigma}}{2} e^{j(\omega t - kz)} + c.c. \]

\[ = \text{Re}(\hat{\sigma} e^{j(\omega t - kz)}) \]

\[ \therefore \text{In phasor form} \]

\[ \nabla \times \vec{E} = -j k \times \vec{E} e^{j(\omega t - k \cdot \vec{r})} \]

take only \text{Re} e^{j\omega t} + c.c.

\[ -j k \times \vec{E} = -j \omega \mu \vec{H} \]

\[ \nabla \times \vec{H} = -j \sigma \vec{E} \]

\[ \nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{E}}{\partial t} \]

gives

\[ -j k \times \vec{H} = \vec{J} + j \omega \vec{E} \]

3.

\[ \gamma + j k x (j k x \vec{E}) = -j \omega \mu (j k x \vec{H}) \]

Sub from 3

\[ + j k x (j k x \vec{E}) = -j \omega \mu (\sigma + j \omega \varepsilon) \vec{E} \]

\[ -k (k \cdot \vec{E}) + k^2 \vec{E} = -j \omega \sigma \vec{E} + \omega^2 \mu \varepsilon \vec{E} \]

0 from Gauss
\[ k^2 = \omega^2 \mu \left( e^{j\gamma} - e^{-j\gamma} \right) \]
\[ k = \beta - j\alpha \quad \text{complex} \]
\[ -\alpha^2 + \beta^2 - 2j\alpha\beta = \omega^2 \mu \epsilon^1 - j\omega^2 \mu \epsilon^{11} \]
\[ \epsilon^1 \quad \text{equal} \quad \epsilon^{11} \quad \text{equal} \]

Equation (7.66) of text.

\[ \eta_c = \sqrt{\frac{\mu}{\epsilon_c}} = \sqrt{\frac{\mu}{\epsilon^1 - j\epsilon^{11}}} \rightarrow \text{intrinsic impedance} \]

\[ \delta = \frac{1}{\alpha} = \text{"skin" depth} \]

If \( \epsilon^{11} \) is large

\[ \alpha = \omega \left\{ \frac{\epsilon^{11}}{2\mu} \right\}^{\frac{1}{2}} \]
\[ = \omega \left\{ \frac{\epsilon^{11}}{2\mu} \right\}^{\frac{1}{2}} = \left\{ \frac{\mu \omega^2}{2} \right\}^{\frac{1}{2}} = \left\{ \frac{\mu \pi f \sigma}{2} \right\}^{\frac{1}{2}} \]

Above \( \alpha \approx \beta \) (\( \epsilon^1 \) small)

\[ \eta_c = \sqrt{\frac{\mu}{\sigma}} = \left( \frac{1 + i}{2} \right) \sqrt{\frac{\mu \epsilon_0}{\sigma}} \]
Surface Impedance

\[ E_x(z) = E_x(0) e^{-j \frac{z}{s}} + \frac{1}{s} e^{j \omega t} \]

\[ j = \sigma E \]

\[ \int j \, dz = \sigma E_x(0) = I_s \, m \]

\[ \frac{1}{s} (1 + j) \]

\( \frac{\sigma}{1 + j} \) = surface impedance

total current is \( I_s \, m = I \)

total voltage is \( E_x l = V \)

\[ \frac{V}{Z_s l} = I \]

\[ Z = \frac{Z_s l}{w} \]
A "wave-guide"

Characteristic of a guided wave

\[ e^{j\omega t - jkz} \]

\[ k = \omega \sqrt{\mu \epsilon} \] but is the same for all \( x \) (plane phase front)

What condition needs to apply

\[ \mathbf{E}_1 e^{j\omega t} \quad e^{-jks\sin \varphi_z - jkcos \varphi_x} \]

\[ \mathbf{E}_2 e^{j\omega t} \quad e^{-jks\sin \varphi_z + jkcos \varphi_x} \]

Thus in going from \( A \) to \( B \)

Wave 1 has phase change \(-k L \sin \varphi_z - k \cos \varphi_x \)

Wave 2 has phase change \(-k L \sin \varphi_z - k \cos \varphi_x \)

If we follow wave 1 just after reflection from \( A \) to \( B \) (with a reflection at \( B' \)) to \( C \) and a final reflection at \( C \), then the total phase change is \( 2k \sin \varphi (2L) + 2k \cos \varphi_a + \Phi_1 + \Phi_2 \) where \( \Phi_1 \) and
\[ \varphi_1, \varphi_2 \text{ are the phase changes upon reflection at B and C respectively.} \]

For a mode to propagate this must reduce to only the phase change due to the Z-component of \( k \) that is \( 2k \sin \theta(a) \). Thus the mode condition is

\[ 2k \sin \theta \alpha + \varphi_1 + \varphi_2 = n (2\pi) \]

where \( n \) is an integer.

For a metal \( \varphi_1 = \varphi_2 = \pi \) so \( \varphi_1 + \varphi_2 \) does not enter. Thus in this case

\[ k \sin \theta = n \frac{\pi}{\alpha} \]

Thus \( \theta \) is quantized

Total field \( \left( \mathbf{E}_z = -\mathbf{E}_x \right) \) by B.C. (Tangential \( E = 0 \) at metal boundary)

\[ \mathbf{E}_y = \mathbf{E}_x e^{-j(\omega t - k_x z)} \]

\[ \mathbf{E}_y = \mathbf{E}_x e^{-j 2\pi \frac{z}{\lambda} \sin \left( \frac{n \pi}{\alpha} \right)} \]

Now we can get \( \mathbf{H}_x \) and \( \mathbf{H}_z \)
easily as well.
\[ H_1 = \frac{E_1}{\sqrt{\mu_0}} \]
\[ H_2 = \frac{E_2}{\sqrt{\mu_0}} \]
\[ E_2 = -E_1 \]

\[ H_1 = H_1 e^{-jk_z z + jk_x x} \]
\[ H_2 = H_2 e^{-jk_z z - jk_x x} \]

\[ H_z = \frac{E_1 e^{-jk_z z - jk_x x}}{\sqrt{\mu_0}} - \frac{E_2 e^{jk_z z + jk_x x}}{\sqrt{\mu_0}} \]

\[ = \frac{E_1}{\sqrt{\mu_0}} \cos \theta 2 \cos (k_x x) \]

\[ H_x = \frac{E_1 e^{-jk_z z - jk_x x}}{\sqrt{\mu_0}} + \frac{E_2 e^{jk_z z + jk_x x}}{\sqrt{\mu_0}} \]

\[ = -\frac{E_1}{\sqrt{\mu_0}} 2j \sin \theta (\sin k_x x) \]

This is a TE_{n} mode, \( H_x = 0 \) at both boundaries and \( k_x a = n \pi \).